Uncertainty Quantification of Offshore Wind Farms Using Monte Carlo and Sparse Grid

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Abstract

The power produced by an offshore wind farm is subject to volatile wind, turbine performance wear and other uncertainties such as availability losses. For the planning stage of wind farms and later for performance improvements of existing wind farms the knowledge about the stochastic distribution of the produced power is very important. It is crucial to understand how uncertainties propagate through the models and ultimately how sensitive the predicted energy production is with respect to these uncertainties.

Due to the multitude of uncertainties, a complete analysis requires very high dimensional numerical integration techniques in order to determine these sensitivities and thus such an analysis has never been done in the literature for the entire set of uncertainties. This work for the first time provides a thorough analysis of all uncertainties modeled by years of data collection and experience from the Vattenfall Europe Windkraft GmbH. To highlight differences in wind park layouts, the analysis is performed on the three wind farms Horns Rev 1, DanTysk and Sandbank. From the total set of nine uncertain parameters, the sensitivity analysis reveals four important candidates, allowing the other parameters to be neglected in future measurement data acquisitions and sensitivity analysis processes. Furthermore, several integration techniques are compared in

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order to provide a recommendation for future projects.

**Keywords:** Offshore wind farm, Uncertainty Quantification, quasi-Monte Carlo, Stochastic collocation

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1. Introduction

The process of turbine induced wake generation is potentially depending on a large set of parameters. Most of these parameters have a high level of uncertainty due to e.g. the impossibility of performing ideal measurements of the volatile wind, the occurrence of material fatigue which influences the performance wear and other uncertainties such as availability losses. In order to make reasonable predictions for the wind farm energy production, it has to be determined how this value varies in relation to changes in these parameters.

In the current literature exists a knowledge gap concerning the sensitivity analysis of a wind farms. This first and foremost includes the investigation of a larger and more complete set of uncertainty parameters and also suitable mathematical methods for their propagation through the entire model.

There have been only few publications dealing with the consideration of uncertainties in parameters for wind farm simulations. Lackner et al. [1] listed a set of wind farm parameters which have an influence on the uncertainty for the prediction of the energy production. Murcia et al. [2] analyzed the influence of measurement uncertainties in some of these parameters in order to check for modeling errors. Foti et al. [3] on the other hand focused on two different uncertainties and directly analyzed their influence on the total power production. Rinker [4] investigated the sensitivity of a single turbine for four parameters which are inflicted with uncertainties. Ashuri et al. [5] investigated the levelized cost of electricity of a single turbine while considering up to four parameters which are inflicted with uncertainties. Padron et al. [6] use polynomial chaos to compute the stochastic effect of wind direction and wind speed on the annual energy production in wind farms.

The novelty of this paper lies in the sensitivity analysis of multiple offshore
wind farms for a large set of uncertainties. We develop a stochastic model to investigate the impact of up to nine uncertain input parameters. Several different numerical integration techniques are investigated, which are used for the propagation of the uncertainty parameters through the model. For the best suitable mathematical method a sensitivity analysis is performed to determine the most influencing uncertainties on the cost function. In a case study we use experienced data about uncertainties from business operation for different wind farm layouts.

The outline of this paper is structured as follows: Section 2 describes deterministic model which we use in order to compute the annual energy production. This involves models for the wake computation, the power generation model and a cost model. In Section 3 the whole model is extended by introducing randomness into the input parameters to get a stochastic model. Section 4 introduces the Uncertainty Quantification methods starting with the classical Monte Carlo method followed by its derivations as the quasi-Monte Carlo and finally the Stochastic Collocation method. The performance of each method will be discussed and finally a recommendation for which method to use based on the presented stochastic model will be given. The stochastic model and the Uncertainty Quantification methods from these two sections are then used in Section 5 where the uncertainty propagation related to different cost functions is investigated. Finally, all results will be summarized in the final Section 6.

2. Deterministic Model for the Annual Energy Production in Wind Farms

The annual energy production (AEP) in wind farms is computed by three sub-models which describe the wind, wake, and power generation in a wind farm. The temporal integration of gross produced power is computed by considering the measurements of wind speed and wind direction within one year. For each wind direction and each wind speed, the power produced by the wind farm needs to be computed where the integral over all directions yields the produced
energy. This value is then used in a cost model to describe economic quantities, see Figure 1. In the following all sub-models are described.

![Diagram of offshore wind farm model]

Figure 1: Structure of the offshore wind farm model. Input parameters are highlighted in green. These parameters are also later used for the uncertainty quantification. Possible outputs of the wind farm model (highlighted in red) are the net annual energy production (AEP) and the levelized cost of electricity (LCOE).

2.1. Wind model

The source of energy for a wind turbine is obviously the wind. Typically, the wind data consists of thousands of measurements for wind direction and wind speed. Figure 2 shows a clustered wind direction distribution over a time period of four years. The distribution is divided into several sectors, standing for the cardinal points and representing the probability of the wind direction by the size of each sector. Furthermore, for each wind direction sector, the distribution of the wind speed is considered independently.

The classical approach is to model the raw data by a Weibull distribution using the maximum likelihood estimation [7],

$$W(u; \lambda, k) = \left( \frac{k}{\lambda} \right) \cdot \left( \frac{u}{\lambda} \right)^{k-1} \cdot \exp \left( - \left( \frac{u}{\lambda} \right)^k \right) \cdot \ell_{\text{wind}},$$

where $\lambda > 0$ is the scale parameter and $k > 0$ is the shape parameter of the Weibull distribution, see Figure 3. Furthermore, we consider wind speed losses $\ell_{\text{wind}}$ due to turbulence, off-yaw axis winds, inclined flow and high shear wind flow. Loss parameters are mainly based on experience and thus are used to make the developed model more realistic, by closing the gap between turbine power curve test conditions and actual conditions at the site.

In the following when referring to the overall model as deterministic, this implies that the input parameters are not subject to any kind of uncertainty.
For the simulation of a wind farm we need to consider each wind direction sector and therein a certain number of wind speeds. It is clear, that the model becomes more precise by increasing the number of wind sectors $N_{\text{dir}}$ and wind speeds $N_{\text{speed}}$.

![Wind direction measurements](image)

Figure 2: Wind direction measurements at the FINO3 research platform, which is placed in the North Sea, about 80 km away from the German island Sylt. The data at 100 m height for eight years from January 2010 to December 2017 is clustered into $N_{\text{dir}}=12, 32, \text{ and } 360$ direction sectors.

![Wind speed measurements](image)

Figure 3: Wind speed measurements at the FINO3 research platform at a height of 100 m for the eight years from January 2010 to December 2017 for the western wind direction sector (255$^\circ$ to 285$^\circ$). The data is fitted by a Weibull distribution using the maximum likelihood estimation [7].

2.2. Wake model

In an offshore wind farm, several wind turbines are arranged in a predefined region. Behind each wind turbine a wake is generated which is responsible for
wind speed reductions for subsequent wind turbines. The task of a wake model is to compute this impeded velocity field behind each wind turbine.

In literature, one can find many different models which describe the wake effect. The first notable wake model was developed by Jensen [8] and Katic et al. [9] in 1986, which is called *PARK model*. Further developments are the *Eddy-Viscosity wake model* [10] and some extensions [11, 12], the *Deep-array wake model* [13], a *linearized Reynolds-averaged Navier–Stokes* model [14] and *Large Eddy Simulations* [15].

In this paper we focus on the *PARK model*, which is still the leading wake model in commercial software tools, because the velocity deficit calculated by the model has a top-hat shape. Furthermore, investigations of Barthelmie et al. [16] show that the computations are quite accurate compared to other more detailed models, e.g., against the $k$-$\varepsilon$ turbulence model which bases on a parabolized Navier–Stokes model [17].

The velocity deficit inside the wake only changes in stream direction. The wake radius grows with a constant factor $k = 0.5/\ln(z/z_0)$, which depends on the hub height $z$ of the turbine and the surface roughness $z_0$. The wake diameter $D_w(x)$ grows linearly by $2k$, see Figure 4. For an inflow velocity $u_0$ the wake velocity $u_w(x)$ at any point inside the wake of a turbine with rotor diameter $D$ is given by

$$u_w(x) = u_0 - \frac{1 - \sqrt{1 - C_t(u_0)}}{1 + 2kD} \cdot \ell_{\text{wake}} u_0 = u_0 - \frac{1 - \sqrt{1 - C_t(u_0)}}{\left(1 + \frac{x}{D \ln(z/z_0)}\right)^2} \cdot \ell_{\text{wake}} \cdot u_0,$$

where the last fraction describes the velocity deficit. The velocity dependent thrust coefficient of the turbine $C_t(u_0)$ is a characteristic property of the turbine type and thus needs to be provided by the manufacturer of the wind turbine, see Figure 5. The parameter $\ell_{\text{wake}}$ is used to consider wake effect losses due to internal turbine arrays, external turbines and future developments in the vicinity of the wind farm.

To compute the incident velocity of a wind turbine, we simply use a weighting factor $\beta$ between the free stream velocity $u_0$ and the wake velocity $u_w(x)$, which
Figure 4: Wind turbine with hub height $z$ and rotor diameter $D$. The wake diameter $D_w$ grows linearly by $2k$. Inside the wake the incident wind speed $u_0$ is reduced to the wake velocity $u_w$. The Figures are taken from [18].

is computed by the circular intersection $A_{\text{Intersection}}$ of the wake cross section with the turbine’s circular area $A_{\text{Turbine}}$, $\beta = \frac{A_{\text{Intersection}}}{A_{\text{Turbine}}}$.

Figure 5: Thrust coefficient $C_t$ and power production of the turbine Vestas V80 with cut-in speed of 4 m/s and cut-out speed of 25 m/s (dashed vertical lines).

2.3. Power generation model

Turbines are converting the wind’s kinetic energy into electrical energy. Thus, the generated power of a wind turbine $P$ depends on the incident wind speed $u$ and the power curve, which is usually provided by the manufacturer, see Figure 5,

\[ P(u) = P_{\text{power\_curve}}(u) \cdot \ell_{\text{power}}. \]  

(3)

With the parameter $\ell_{\text{power}}$, power curve losses as material performance deviations from the expected power curve are considered. The power curve depends on a cut-in speed $u_{\text{cutin}}$ and a cut-out speed $u_{\text{cutout}}$ which specify the range of
wind speed in which the turbine generates power. If the wind speed is lower than \( u_{\text{cutin}} \) there is not enough wind for efficient power production, and for wind speeds larger than \( u_{\text{cutout}} \) the wind is too strong such that the turbine may be damaged.

The gross annual energy production is given as the produced power for the duration of one year (in hours), thus

\[
E_{\text{AEP\text{\textregistered}gross}} = (8760h + 6h) \cdot P, \tag{4}
\]

while the total generated power \( P \) is computed from the generated power for each wind direction \( \varphi \),

\[
P := \int_{0}^{2\pi} P_{\varphi} d\varphi \approx \sum_{i=1}^{N_{\text{dir}}} w_{\varphi_i} \cdot P_{\varphi_i}, \tag{5}
\]

with direction \( \varphi_i \), weight \( w_{\varphi_i} \) of the quadrature rule. The power \( P_{\varphi_i} \) for one wind direction is given by integrating along the corresponding probability function and the power curve.

### 2.4. Cost model

As a result of the last three sub-models we can compute a value for the gross annual energy production by integrating for each wind direction and wind speed, computing the wake velocity for subsequent turbines and evaluating the power curve, see Figure 1. This value is now used to compute different economic indicator functions, e.g. the net annual energy production, the levelized cost of electricity [1], the net present value [19], or the internal rate of return [20]. In this work, we consider the following two economic indicator functions.

- **The net annual energy production,**

\[
E_{\text{AEP\text{\textregistered}net}} = E_{\text{AEP\text{\textregistered}gross}} \cdot \ell_{\text{performance}}, \tag{6}
\]

is the basic value for all other economic indicator functions. With the plant performance loss \( \ell_{\text{performance}} \), we consider electrical losses due to availability of turbines, high wind hysteresis and environmental performance degradation, such as icing and high temperatures.
• The levelized cost of electricity given by Lackner and Elkinton [1]

\[
K_{\text{LCOE}} = \frac{C_{\text{capital}} \cdot \frac{(1 + r_{\text{rate}})^T}{(1 + r_{\text{rate}})^T - 1} + C_{\text{O&M}}}{E_{\text{AEPnet}}},
\]

with total installed capital costs \(C_{\text{capital}}\) for turbines, cabling, substation, decommission etc., annual operation and maintenance costs \(C_{\text{O&M}}\), and discount rate \(r_{\text{rate}}\) including debt, taxes and insurance over the time period \(T\).

3. Stochastic Model for the Annual Energy Production in Wind Farms

The models described in Section 2 are purely deterministic. They will always compute the same results for the same inputs. But to consider disturbances in some parameters we need to introduce some random variables and include them into the model to finally get a stochastic model.

3.1. Uncertain parameters

The deterministic model from Section 2 is extended considering uncertainties in the parameters. In [21] several uncertainty inflicted parameters have been identified, see Table 1. To consider the uncertain disturbance for input parameters, the input parameters are now modeled as random variables \(\xi_i\). In this work we are modeling all random variables as independent and normally distributed \(\xi_i \sim \mathcal{N}(\mu_i, \sigma_i)\), whose mean is centered around the original undisturbed value \(\mu_i\). We want to stress, that this is a modeling assumption and might not be justified for some of the presented uncertainties. The economic-based random variables are almost surely independent from the ones from the physical models and are also independent from each other as they all address separate topics. The same holds for most of the uncertain variables from the physical domain. The uncertainty in the thrust coefficient \(C_t\) for example is considered to be a geometric design property of the turbine blades and thus the uncertainty lies within the generated thrust and not the static pressure on the blades. Similar holds for the power curve where the uncertainty is modeled in
the mapping of the wind velocity to the produced power and thus is independent from the velocity itself. Thus, for the independence of uncertain parameters we only assume the independence of the uncertainty on the surface roughness and the uncertainty in the measured wind speed.

Each random variable can be rewritten to a multiplication between the undisturbed value and a normal distribution $\mathcal{N}(1, \sigma)$ with the mean value of one. As the standard deviation $\sigma$ is a relative property, it can be chosen to resemble any observed deviation $d$. In this work, the deviation of the measurement data $d$ describes the exceedance probability of 90% and therefore $\sigma$ can be computed by evaluating the inverse of its distribution function for which, in case of the normal distribution, 90% of the data lies below the bell curve. The following uncertain parameters are considered:

- **Wind speed** due to site measurements, historic wind resource or the measure-correlate-predict method, vertical extrapolation, future variability, and wind flow extrapolation.
- **Wake effect** due to model inaccuracies and neighboring sites.
- **$C_t$ curve** due impacts of atmospheric stability and site conditions for which $C_t$ curve is not valid.
- **Surface roughness** due to changing surface conditions caused by weather or tides.
- **Power curve** due to impacts of atmospheric stability, and site conditions for which power curve is not valid.
- **Plant performance** due to electrical efficiency, availability of turbines, internal and external grid, due to environmental as blade soiling, blade degradation and weather effects.
- **Capital costs** due to steel price fluctuations.
- **Annual O&M costs** due to use of new technologies.
- **Discount rate** due to fluctuations of the economic discount rate.
3.2. **Stochastic model**

The above defined random variables are now included into the model to finally get a stochastic model.

**Wind.** Due to inaccurate measurements, imprecise long-term predictions, interannual variability, and further interferences [21], the distribution of the wind speed \( u \) is a highly uncertain parameter. Therefore, we disturb the raw data of the wind speed with a normally distributed random variable \( \xi_{\text{wind}} \) such that disturbed probability density functions of the Weibull distribution are obtained, see Figure 6. In [20] and in [22] it is shown that a disturbance \( d \) of the wind speed corresponds to the disturbance \( d \) of the Weibull parameter \( \lambda \), such that the resulting probability for each wind speed \( u \) can be formulated as random variable, compare with equation (1):

\[
W(u; \lambda, k, \xi_{\text{wind}}) = \left( \frac{k}{\lambda \cdot \xi_{\text{wind}}} \right) \cdot \left( \frac{u}{\lambda \cdot \xi_{\text{wind}}} \right)^{k-1} \cdot \exp \left( - \left( \frac{u \cdot \lambda}{\lambda \cdot \xi_{\text{wind}}} \right)^k \right) \cdot \ell_{\text{wind}},
\]

where \( \lambda > 0 \) is the scale parameter and \( k > 0 \) is the shape parameter of the Weibull distribution which are determined with the maximum likelihood estimation using the undisturbed wind speed data.

**Wake.** Within a wind farm or outside of the wind farm there can be some wake effects from future installations. The predicted wake effect is disturbed due to

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<th>Uncertain parameters</th>
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<th>( \xi_{\text{rough}} )</th>
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Table 1: Uncertain parameters of the wind farm model.
uncertainty in the model inputs (including wind direction), and that associated with model performance and appropriateness for site. Furthermore we need to include uncertainty related to any proposed neighboring sites (construction timing, layout, turbine type) [21]. Therefore we disturb the velocity deficit of a wake with a normally distributed random variable \( \xi_{\text{wake}} \). Furthermore, the wake model depends on the \( C_t \) curve and the surface roughness \( z_0 \). Due to imprecise measurements of the \( C_t \) curve, we disturb this parameter with the a random variable \( \xi_{\text{ct}} \), see Figure 7. Here, in order to prevent negative values in the root of the numerator of equation 9, we used a truncated normal distribution, which enforces \( C_t(u_0) \cdot \xi_{\text{ct}} \leq 1 \) as this could otherwise lead to non-physical complex wake velocities.

The surface roughness depends on the topography and flora [23], i.e. for offshore wind farms it depends on the wave field, wind speed, upstream fetch and water depth [24]. Thus, also this parameter \( z_0 \) should be stochastic and is therefore disturbed with the a normally distributed random variable \( \xi_{\text{rough}} \). Altogether, the wind speed behind a turbine at any point \( x \) from equation (2)
changes as follows:
\[
\tilde{u}_w(x, \xi_{\text{wake}}, \xi_{\text{ct}}, \xi_{\text{rough}}) = u_0 - \frac{1 - \sqrt{1 - C_t(u_0) \cdot \xi_{\text{ct}}}}{(1 + D \cdot \ln(z/(z_0 \cdot \xi_{\text{rough}})))^2} \cdot \ell_{\text{wake}} \cdot \xi_{\text{wake}} \cdot u_0. \tag{9}
\]

*Power generation.* In the power generation model the turbine performance is disturbed due to material fatigue which leads to uncertainty in the power curve. Furthermore, there is uncertainty on performance under site conditions for which the power curve is not valid. This also includes the impact of atmospheric stability and uncertainty associated with uncertain icing losses as well as other environmental losses, e.g. blade soiling, blade degradation, weather effects. As it is difficult to quantify these sources of uncertainty in terms of individual standard deviations, we model them as a single uncertain parameter and thus disturb the power curve by the normally distributed random variable \(\xi_{\text{power}}\) (see Figure 7). This changes equation (3) for the power curve as follows:
\[
\tilde{P}(u, \xi_{\text{power}}) = P_{\text{power\_curve}}(u) \cdot \ell_{\text{power}} \cdot \xi_{\text{power}}. \tag{10}
\]

Figure 7: Exemplary disturbed \(C_t\) and power curve of a Vestas V80 turbine with a cut-in speed of 4 m/s and cut-out speed of 25 m/s (dashed vertical lines). The red tube illustrates the \(C_t\) and power curve with a disturbance of \(\pm 10\%\).

*Net annual energy production.* The *net annual energy production* is the basic value for all other economic indicator functions. As the plant performance losses due to e.g. availability or curtailment are uncertain, we disturb the performance with the normally distributed random variable \(\xi_{\text{performance}}\), which results into the following formula:
\[
\tilde{E}_{\text{AEP\_net}} = E_{\text{AEP\_gross}} \cdot \ell_{\text{performance}} \cdot \xi_{\text{performance}}. \tag{11}
\]
Levelized cost of electricity. The capital costs $C_{\text{capital}}$ mainly depend on the price of steel. But because of the long planning stage of several years for a wind farm the calculation depends on long-term predictions for the steel price which is very volatile. Therefore, we disturb the capital costs with a normally distributed random variable $\xi_{\text{capital}}$. The same argument holds for the discount rate $r_{\text{rate}}$, which is in an early planning stage very unsure such that this parameter is disturbed with a normally distributed random variable $\xi_{\text{rate}}$. The costs for annual operation and maintenance $C_{\text{O&M}}$ also underlie the volatile behavior of the price of steel (for the material), and other political decisions like payroll taxes. Therefore, also this parameter is disturbed with a normally distributed random variable $\xi_{\text{o&m}}$. Altogether, the the levelized cost of electricity from (7) changes as follows:

$$\tilde{K}_{\text{LCOE}} = C_{\text{capital}} \cdot \xi_{\text{capital}} \cdot \frac{(1 + \tilde{r}_{\text{rate}})^T \cdot \tilde{r}_{\text{rate}}}{(1 + r_{\text{rate}})^T - 1} + C_{\text{O&M}} \cdot \xi_{\text{o&m}} \cdot E_{\text{AEPnet}},$$

with disturbed discount rate $\tilde{r}_{\text{rate}} = r_{\text{rate}} \cdot \xi_{\text{rate}}$.

4. Methods of Uncertainty Quantification

As is it the goal of this paper to investigate the influence of uncertainties in the input parameters onto different model outputs, e.g. gross annual energy production, it is necessary to introduce methods which allow to quantify their influence on a certain quantity of interest. The overall idea of propagating uncertainties through models and then computing the sensitivities of these model in relation to the inputs is in the literature referred to as Uncertainty Quantification (UQ).

From a practical point of view the methods for Uncertainty Quantification ([25], [26], [27], [28]) can be divided into two different categories: While intrusive methods introduce changes to the original problem as the governing equations become statistical, non-intrusive methods on the other hand evaluate the original problem with varying inputs and compute the statistics from these computed results. For complex problems, a non-intrusive method is often
favored as it requires no modification to the original code. While Lackner et al. [1] and Foti et al. [3] used a Monte Carlo simulation, Murcia et al. [2] performed a Latin Hypercube simulation for their investigations.

In this paper the influence of the uncertainty of some parameters in the context of an offshore wind farm will be investigated by the means of the classical Monte Carlo [25] method, low-discrepancy methods quasi-Monte Carlo [29] and also the Stochastic Collocation [30] method. We will then provide a recommendation for the method which performs best with respect to the underlying offshore wind farm problem setting.

Figure 8: Visual representation of different sampling strategies on a two-dimensional unit square. (a) classical Monte Carlo with pseudo random numbers generated by the Mersenne Twister Engine from the C++ standard library, (b) quasi-Monte Carlo with pseudo random numbers generated by the Sobol sequence, and (c) Stochastic Collocation on Smolyak sparse grids with Clenshaw Curtis nodes.

4.1. Monte Carlo

In order to compute a given quantity of interest, such as for example the expectation of the annual energy production, the problem can mathematically be written as:

$$E[u(\vec{X}, \vec{\xi})] = \int_{\Gamma} u(\vec{X}, \vec{\xi}) \rho(\vec{\xi}) d\vec{\xi},$$

where $u(\vec{X}, \vec{\xi})$ is the disturbed model involved in computing the annual energy production, $\vec{\xi}$ are the random variables modeling uncertainties within the inputs with $\rho(\vec{\xi})$ as the respective probability density function and $\vec{X}$ the vector
of undisturbed parameters.

The Monte Carlo method (MC) uses sampling in the probability space of the associated random variable $\vec{\xi}$ to evaluate the integral in equation 13. By computing $M$ deterministic solutions (sampling), each starting from a different set of realizations of the uncertain parameters, $M$ solutions of the type $u^m(\vec{X}) = u(\vec{X}, \vec{\xi}^m)$ are obtained. If $\vec{\xi}^m$, $m = 1, \ldots, M$ is a sequence of independent and identically distributed random variables, application of the central limit theorem yields:

$$\frac{1}{M} \sum_{m=1}^{M} u(\vec{X}, \vec{\xi}^m) \xrightarrow{a.s.} \mathbb{E}[u(\vec{X}, \vec{\xi})]$$

This means that, in the limit, the method converges to a fixed value for the mean and also the variance of the Quantity of Interest. The rate of convergence for the Monte Carlo method with random sampling is $O(M^{-1/2})$, as shown by Caflisch [31]. $O(\cdot)$ in this context describes the upper bound for the growth rate of a function. Therefore, in order to achieve one additional digit of accuracy, it is necessary to compute 100 times more samples. This slow convergence rate can cause problems in case of computationally expensive problems. On the other hand, the convergence rate of the Monte Carlo sampling is not a function of the dimension of the probability space.

Generating random numbers for the sampling process is a difficult task in practice as computers are deterministic machines. In this work we use pseudo random numbers generated by the Mersenne Twister method [32] from the C++ standard library. A visual representation of generated samples can be seen in Figure 8a.

4.2. Quasi-Monte Carlo

An improvement to the classical Monte Carlo method is the quasi-Monte Carlo method (QMC). It relies on the sample principle like the classical Monte Carlo method with the difference being, that it makes use of a low-discrepancy sequence in order to generate its random numbers. Morokoff and Caflisch [33] examined three different low discrepancy sequences: the Halton, Sobol and
Faure sequence. The result indicated that Halton sequences are best for up to six dimensions and the Sobol sequence is best for all higher dimensions. As we are interested in these high dimensional cases, the Sobol sequence is used in this paper. The Sobol sequence can briefly be explained as a sequential instruction set that fills a multi-dimensional hypercube, while trying to avoid the creation of void regions. These created values are deterministic and thus are called pseudo-random, but evenly fill the hypercube and therefore potentially lead to a faster convergence compared to the pure Monte-Carlo method. The more equal filling can be seen in Figure 8b. For a detailed explanation regarding the generation of the sequence, see Bratley and Fox [34]. By using the Sobol sequence to generate pseudo random numbers, the convergence of the quasi-Monte Carlo is of the order $O(M^{-1}(\log M)^{\text{dim}(\vec{r})})$ [31]. This means that for small dimensions, the quasi-Monte Carlo simulation only needs to compute roughly five to ten times fewer samples in order to achieve one additional digit of accuracy compared to the classical Monte Carlo method.

4.3. Sparse Grid Stochastic Collocation

Compared to the previous methods, the sparse grid Stochastic Collocation (SC) method is not a variant of the classical Monte Carlo method. The main idea of the sparse grid Stochastic Collocation method is to choose a set of $M$ collocations points in probability space and then to compute the solution at these points. As the positions of the collocation points are generally free to choose, the sparse grid Stochastic Collocation method selects them based on a quadrature rule and exploits the corresponding quadrature weights to compute the statistics of a given quantity of interest such as e.g. the annual energy production. In case of a high dimensional probability space it is thus necessary to use an efficient quadrature rule, as for common methods the number of required quadrature points increases exponentially with the dimension of the probability space. This leads to overall expensive computations as a solution of the deterministic problem needs to be computed on every quadrature point. One way to construct such an efficient rule is by using the so-called Smolyak
sparse grids. These grids are constructed from nested one-dimensional quadrature rules, which restricts the degrees of freedom involved in the discretization of the problem and thus allows for a slower growth of the required quadrature points. In this paper Smolyak sparse grids with Clenshaw Curtis nodes are used for the computations involving the sparse grid Stochastic Collocation method. As this node type yields a quadrature set for the bounded interval \([-1, 1]\) and thus would not be a suitable quadrature rule for unbounded probability density functions as e.g. the normal distribution, we use an inverse cumulative distribution function transform to expand the quadrature set to \((−∞, ∞)\) as described in [35]. For further details on the sparse grid method, see [25, 36]. Figure 8c shows an example for such a sparse grid for the domain \([0, 1]^2\).

The convergence rate of the method is of the order \(O(M^{-\alpha}(\log M)^{(\dim(\mathcal{G})-1)(\alpha+1)})\), with dimension of the uncertain parameter space \(d\), \(M\) as total number of grid points and \(\alpha\) depending on the regularity of the solution. It has to be stressed that the number of samples \(M\) is indirectly determined by the dimensionality of the probability space and the level of the underlying quadrature rule which can be selected by the user. Also differing from the previously discussed Monte Carlo type methods, is the convergence rate of the sparse grid Stochastic Collocation method as it decreases for higher dimensions. This phenomenon is often referred to as the ”curse of dimensionality” and restricts usability of collocation methods for high dimensional probability spaces.

4.4. Error comparison of UQ methods

In order to demonstrate the convergence behavior of the described methods, we use the computation of the levelized cost of electricity (see Figure 1) as a test case as for this benchmark. The uncertain parameters in this computation resemble the maximal amount of parameters in the results presented in Section 5. The associated probability distributions used in the sampling process for each parameter can be seen in Table 4, while the remaining input data is configured according to the Horns Rev 1 wind farm dataset. As we are computing the levelized cost of electricity, all nine parameters from the Table have an influence
on the solution and therefore the resulting probability space is nine-dimensional.

The convergence rates of all presented methods can again be seen in Table 2. As these values only show the theoretical order and neglect the influence of any constant factors, Figures 9 and 10 show the numerically studied error evolutions. This is especially interesting for the sparse grid Stochastic Collocation method, as its convergence rate depends on the unknown smoothness $\alpha$ of the underlying problem. The methods are compared in terms of the relative error in the expectation and variance with respect to a quasi-Monte Carlo simulation computed with a sufficiently large sample size of $M = 1 \cdot 10^8$. Normally the classical Monte Carlo simulation should be used as a reference as it will always converge towards the correct result for $M \to \infty$, but the high dimensionality of the problem combined with the methods slow convergence rate demanded an unreasonable sample size.

<table>
<thead>
<tr>
<th>Monte Carlo</th>
<th>quasi-Monte Carlo</th>
<th>Sparse Grid Stochastic Collocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{O}(M^{-1/2})$</td>
<td>$\mathcal{O}(M^{-1} (\log M)^{\dim(\xi)})$</td>
<td>$\mathcal{O}(M^{-\alpha} (\log M)^{\dim(\xi) - 1} (\alpha + 1))$</td>
</tr>
</tbody>
</table>

Table 2: Order of the convergence rates of the presented UQ methods.

Figure 9: Convergence behavior of the presented methods in terms of the relative error in the mean w.r.t to the quasi-Monte Carlo reference solution.

The presented results show that the Monte Carlo is the least accurate method in terms of the observed error across all sample sizes which coincides with its
Figure 10: Convergence behavior of the presented methods in terms of the relative error in the variance w.r.t to the quasi-Monte Carlo reference solution.

Theoretical convergence rate. The sparse grid Stochastic Collocation method shows a faster as well as a much smoother error decline compared to the quasi-Monte Carlo method, despite having a higher initial error for lower grid levels. Both methods achieve similar relative errors of $7 \cdot 10^{-7}$ for the expectation and $9 \cdot 10^{-5}$ for the variance with respect to the reference quasi-Monte Carlo solution. The dashed lines in Figure 9 and 10 show the theoretical convergence rates of the classical Monte Carlo and the quasi-Monte Carlo methods and while the classical Monte Carlo method shows the expected convergence rate, it is worth noting that the convergence rate for the quasi-Monte Carlo method is plotted with $\dim \xi^* = 1$ and even though the dimensionality of the problem is much higher, the quasi-Monte Carlo shows a comparable error decline, hinting that the probability space of the underlying problem might be dominated by a single dimension and that the low discrepancy sequence of the method can successfully exploit this fact for an accelerated convergence behavior. This observation becomes more apparent in the sensitivity analysis presented in Section 5.

Considering the computational costs of both methods, the costs of the sparse grid Stochastic Collocation method scale in a nonlinear way with the dimension of the probability space and the grid level, while the sequence generation of the quasi-Monte Carlo method is linear in the number of sample points and
thus potentially faster for large numbers of samples/dimensions. As the grid calculations could also be done once and stored prior to the actual Uncertainty Quantification analysis, this does not pose a significant drawback except for very high dimensions and grid levels.

Based on the obtained results, it can be concluded that the quasi-Monte Carlo method is favorable over the other methods. This observation is based on the three criteria: error control, computational overhead, and finally ease of implementation. While Stochastic Collocation and quasi-Monte Carlo yield comparable results for the demonstrated sample sizes, quasi-Monte Carlo allows for any desired sample size and thus a good error control, while the used sparse grids, due to their nested node property, only allow for very discrete sample sizes, which also exponentially increase with the selected level (see marker in the Stochastic Collocation plots in Figures 9 and 10). Furthermore, quasi-Monte Carlo does not required any precomputations as computing the next sample from the Sobol sequence has negligible computational cost. Lastly, do numerical codes for the Sobol sequence have a better availability across multiple programming languages compared to arbitrary dimension and level sparse grid generators.

The presented results in Section 5 are therefore solely obtained by the application of the quasi-Monte Carlo method. For the sample size we choose $1 \cdot 10^6$ samples as the obtained results show a relative error of $O(10^{-6})$ for the LCOE, which also includes the computation of the AEP (see Figure 1) and thus errors for the mean should be of the order $O(1) [\text{GWh}]$ for all presented test cases.

5. Case Studies

This Section deals with the evaluation of the stochastic models introduced in Section 3 by using the methods of Uncertainty Quantification from Section 4. The influence of the input variables on the computed net annual energy production and levelized cost of electricity will be determined, and their computed statistics will be analyzed.
For the model we will use the parameters for the three wind farms Horns Rev 1, DanTysk and the Sandbank. As each of these wind farms differ significantly in the used turbine types and grid layout, they will also have different losses and sensitivities with respect to the presented uncertainties. Because the wind farms are all relatively closely positioned to each other, the same wind data will be used in each of the evaluations. The source of this wind data is the FINO3 met mast, positioned about 50 km southwest of Horns Rev, 2 km west from DanTysk, and 20 km east from Sandbank. We use all data recorded between January 2010 and December 2017 at a height of 100 meters.

For each wind farm, turbine positions are given in Figure 11, its turbine $C_t$ and power curve are shown in Figure 12, its general settings, losses and uncertainties are listed in the Tables 3 and 4. The values presented in these tables originate from data which was gathered over multiple years by the Vattenfall Europe Windkraft GmbH.

Before analyzing the propagation of uncertainties, we introduce some of the notation used by the variance-based sensitivity analysis, as described by Saltelli et al. [40]. This notation is especially designed to express how the uncertainty in the model output can be linked to uncertainties in the inputs. The previously derived stochastic model will in the following be resembled by $Y$, while the model inputs will be referred to as $\xi_i$. The uncertainty propagation of $\xi_i$ through $Y$ is best described in the context of this notation by the sensitivity measure [40]

$$S_i = \frac{\text{var}_\xi(E_{\xi_i}(Y|\xi_i))}{\text{var}(Y)},$$

(14)

which is technically the first order sensitivity coefficient that measures e.g. the

---

1More information about the Horns Rev 1 wind farm can be found online: https://powerplants.vattenfall.com/horns-rev
2More information about the DanTysk wind farm can be found online: https://powerplants.vattenfall.com/dantysk
3More information about the Sandbank wind farm can be found online: https://powerplants.vattenfall.com/sandbank
4More information about the FINO3 met mast can be found online: http://www.fino3.de/en/
Figure 11: Turbine positions for the wind farms Horns Rev 1 (left), DanTysk (center), and Sandbank (right). The constructed wind farm Horns Rev 1 is placed in the North Sea close to the Danish coast. This wind farm has been built in 2002 and is commonly used as a test case for all kinds of offshore wind farm related research, see e.g. [2], [37], [38]. The DanTysk wind farm is located close to the German shore in the North Sea, approximately 70 km western of the island Sylt. Built in 2014, it covers an area of 70 square meters and can theoretically produce 288 MW of power, if each of its 80 turbines would operate at full load. The Sandbank wind farm is located right next to the DanTysk wind farm and is the newest of all presented wind farms as it was just recently built in 2017. Containing 72 turbines, laid out in rows across an area of 60 square kilometers, each capable of producing 4 MW of electrical power, the wind farm has a theoretical peak electrical power output of 288 MW. The turbines also have a much higher cut-out speed compared to the turbines of the other two wind farms.

*additive effect of $\xi_i$ on the model output. $S_i$ can also be interpreted in terms of expected reduction of variance. This interpretation allows an easier understanding of the factors involved in the computation of $S_i$:*

- $\text{var}(Y)$: variance of the output with all inputs modeled as random variables
- $\text{var}_{\xi_i}(E_{\xi_{-i}}(Y|\xi_i))$: expected reduction in variance that would be obtained if $\xi_i$ could be fixed

In the following figures we use box plots to visualize statistical characteristics of the solution. The inside of the box, that is limited by the lower and upper quartile, represents 50% of the computed samples. The vertical line inside the
Figure 12: Thrust coefficient $C_t$ (left) and power production (right) of the turbines Vestas V80 [39], Siemens SWT-3.6-120, and Siemens SWT-4.0-130. The dashed vertical lines show the cut-in and cut-out speeds.

box stands for the median of the data set. The lower whisker is the smallest data value which is larger than lower quartile $-1.5 \cdot$ inter-quartile-range, where the “inter–quartile–range” is the difference between upper quartile and lower quartile. The upper whisker is the largest data value which is smaller than upper quartile $+1.5 \cdot$ inter-quartile-range.

5.1. Net annual energy production

In Figure 13 the results regarding the net annual energy production for all six influencing uncertainty parameters, i.e. wind speed, wake effect, $C_t$ curve, surface roughness, power curve and power loss are shown. For their deviation we choose two test cases: an academic test case for which we choose equal deviations of 5% for all six uncertainty inflicted parameters, while for the realistic test case we choose the deviations as given in Table 4 by the Vattenfall Europe Windkraft GmbH.

Each box plot represents the results of an uncertain parameter. The topmost box plot represents the model output for the case that the six uncertainties are disturbed at the same time. In the box plot it can be seen that the deviation from the expected net annual energy production value behaves similarly in both directions of the net AEP. The deviations of the wind speed, power curve and plant performance have the largest impact on the net annual energy production in comparison to all other uncertainties. This holds for the academic and realistic test case.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Horns Rev 1</th>
<th>DanTysk</th>
<th>Sandbank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Positions</td>
<td>see Figure 11</td>
<td>see Figure 11</td>
<td>see Figure 11</td>
</tr>
<tr>
<td>Wind speed losses</td>
<td>$\ell_{\text{wind}}$</td>
<td>98.5 %</td>
<td>99.2 %</td>
</tr>
<tr>
<td>2 Turbine type</td>
<td>Vestas V80-2.0MW</td>
<td>Siemens SWT-3.6-120</td>
<td>Siemens SWT-4.0-130</td>
</tr>
<tr>
<td>Rotor diameter</td>
<td>D</td>
<td>80 m</td>
<td>120 m</td>
</tr>
<tr>
<td>Hub height</td>
<td>z</td>
<td>70 m</td>
<td>88 m</td>
</tr>
<tr>
<td>Cut-in speed</td>
<td>$u_{\text{cutin}}$</td>
<td>4 m/s</td>
<td>4 m/s</td>
</tr>
<tr>
<td>Cut-out speed</td>
<td>$u_{\text{cutout}}$</td>
<td>25 m/s</td>
<td>32 m/s</td>
</tr>
<tr>
<td>Surface roughness</td>
<td>$z_0$</td>
<td>$0.2 \cdot 10^{-3}$ m</td>
<td>$0.2 \cdot 10^{-3}$ m</td>
</tr>
<tr>
<td>Wake effect losses</td>
<td>$\ell_{\text{wake}}$</td>
<td>99.9 %</td>
<td>99.9 %</td>
</tr>
<tr>
<td>3 Power curve</td>
<td>$P(u)$</td>
<td>see Figure 12</td>
<td>see Figure 12</td>
</tr>
<tr>
<td>$C_t$ curve</td>
<td>$C_t(u)$</td>
<td>see Figure 12</td>
<td>see Figure 12</td>
</tr>
<tr>
<td>Power curve losses</td>
<td>$\ell_{\text{power}}$</td>
<td>98.8 %</td>
<td>98.9 %</td>
</tr>
<tr>
<td>4 Total capital costs</td>
<td>$C_{\text{capital}}$</td>
<td>278 000 000 €</td>
<td>1 000 000 000 €</td>
</tr>
<tr>
<td>Annual operation and maintenance costs</td>
<td>$C_{\text{O&amp;M}}$</td>
<td>52 000 000 €</td>
<td>43 200 000 €</td>
</tr>
<tr>
<td>Discount rate</td>
<td>$r_{\text{rate}}$</td>
<td>2 %</td>
<td>3.75 %</td>
</tr>
<tr>
<td>Project lifetime</td>
<td>$T$</td>
<td>20 years</td>
<td>20 years</td>
</tr>
<tr>
<td>Plant performance losses</td>
<td>$\ell_{\text{performance}}$</td>
<td>88.5 %</td>
<td>88.8 %</td>
</tr>
</tbody>
</table>

Table 3: Parameters for the wind farms Horns Rev 1, DanTysk, and Sandbank.

The results in terms of the variance-based sensitivity analysis are shown in Table 5 (academic test case) and Table 6 (realistic test case). In the realistic test case, for all three wind farms the sensitivity value is higher than 84%. The sensitivities of the power curve and plant performance follow with about 3 to 9%. All remaining uncertainties (wake effect, $C_t$ curve and surface roughness) are below 0.3% and thus have a rather negligible influence on the variance of the model output.
Figure 13: Graphical visualization of the sensitivity analysis with all six uncertainty parameters wind speed, wake effect, $C_t$ curve, surface roughness, power curve and plant performance at the same time (simultaneously) compared to single disturbance of the uncertainty parameters computed with the quasi-Monte Carlo method with 100,000 samples for the Horns Rev 1, DanTysk and Sandbank wind farms. The mean AEP corresponds to the given values in literature, namely 599.5 GWh (Horns Rev 1), 1310.6 GWh (DanTysk), 1316.4 GWh (Sandbank).
### Table 4: Uncertain parameters for the wind farms Horns Rev 1, DanTysk, and Sandbank.

The surface roughness parameter is given by using the maximum approximation of Foti et al. [3], \( \sigma_{\text{rough}} = 1.5 \cdot 10^{-5}/z \). The values presented in this table originate from data which was gathered over multiple years by the Vattenfall Europe Windkraft GmbH.

<table>
<thead>
<tr>
<th>Uncertainty</th>
<th>Horns Rev 1</th>
<th></th>
<th>DanTysk</th>
<th></th>
<th>Sandbank</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>deviation</td>
<td>( \sigma )</td>
<td>deviation</td>
<td>( \sigma )</td>
<td>deviation</td>
<td>( \sigma )</td>
</tr>
<tr>
<td>1 Wind speed</td>
<td>8.0%</td>
<td>0.0486</td>
<td>6.0%</td>
<td>0.0365</td>
<td>7.5%</td>
<td>0.0456</td>
</tr>
<tr>
<td>2 Wake effect</td>
<td>3.0%</td>
<td>0.0182</td>
<td>3.0%</td>
<td>0.0182</td>
<td>3.0%</td>
<td>0.0182</td>
</tr>
<tr>
<td>( C_t ) curve (truncated)</td>
<td>2.0%</td>
<td>0.0122</td>
<td>2.0%</td>
<td>0.0122</td>
<td>2.0%</td>
<td>0.0122</td>
</tr>
<tr>
<td>Surface roughness</td>
<td>0.000021%</td>
<td>1.277 \cdot 10^{-5}</td>
<td>0.000017%</td>
<td>1.034 \cdot 10^{-7}</td>
<td>0.000016%</td>
<td>9.737 \cdot 10^{-8}</td>
</tr>
<tr>
<td>3 Power curve</td>
<td>2.0%</td>
<td>0.0122</td>
<td>2.0%</td>
<td>0.0122</td>
<td>2.0%</td>
<td>0.0122</td>
</tr>
<tr>
<td>4 Plant performance</td>
<td>2.3%</td>
<td>0.0140</td>
<td>2.1%</td>
<td>0.0128</td>
<td>2.1%</td>
<td>0.0128</td>
</tr>
<tr>
<td>Capital costs</td>
<td>3.0%</td>
<td>0.0182</td>
<td>3.0%</td>
<td>0.0182</td>
<td>3.0%</td>
<td>0.0182</td>
</tr>
<tr>
<td>Annual O&amp;M costs</td>
<td>3.0%</td>
<td>0.0182</td>
<td>3.0%</td>
<td>0.0182</td>
<td>3.0%</td>
<td>0.0182</td>
</tr>
<tr>
<td>Discount rate</td>
<td>4.0%</td>
<td>0.0234</td>
<td>1.0%</td>
<td>0.0061</td>
<td>0.5%</td>
<td>0.0030</td>
</tr>
</tbody>
</table>
### Table 5: Results of variances based sensitivity analysis where all uncertainties are fixed to 5% of net AEP calculation with 100,000 samples using the quasi-Monte Carlo Method.

<table>
<thead>
<tr>
<th>Uncertainty</th>
<th>Horns Rev 1</th>
<th>DanTysk</th>
<th>Sandbank</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>variance</td>
<td>sensitivity</td>
<td>variance</td>
</tr>
<tr>
<td>1 Wind speed $\xi_{\text{wind}}$</td>
<td>1.6987 · 10^9</td>
<td>48.12 %</td>
<td>5.5619 · 10^9</td>
</tr>
<tr>
<td>2 Wake effect $\xi_{\text{wake}}$</td>
<td>1.1276 · 10^7</td>
<td>0.32 %</td>
<td>3.125 · 10^7</td>
</tr>
<tr>
<td>$C_{t}$ curve $\xi_{\text{ct}}$</td>
<td>2.1235 · 10^7</td>
<td>0.61 %</td>
<td>5.1429 · 10^7</td>
</tr>
<tr>
<td>Surface roughness $\xi_{\text{rough}}$</td>
<td>4.2003 · 10^6</td>
<td>0.12 %</td>
<td>1.7947 · 10^7</td>
</tr>
<tr>
<td>3 Power curve $\xi_{\text{power}}$</td>
<td>9.0764 · 10^8</td>
<td>25.86 %</td>
<td>4.3241 · 10^9</td>
</tr>
<tr>
<td>4 Plant performance $\xi_{\text{performance}}$</td>
<td>8.9968 · 10^9</td>
<td>25.63 %</td>
<td>4.2771 · 10^9</td>
</tr>
</tbody>
</table>

### Table 6: Results of variances based sensitivity analysis of net AEP calculation with 100,000 samples using the quasi-Monte Carlo Method.

<table>
<thead>
<tr>
<th>Uncertainty</th>
<th>Horns Rev 1</th>
<th>DanTysk</th>
<th>Sandbank</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>variance</td>
<td>sensitivity</td>
<td>variance</td>
</tr>
<tr>
<td>1 Wind speed $\xi_{\text{wind}}$</td>
<td>1.5966 · 10^9</td>
<td>92.81 %</td>
<td>2.9557 · 10^9</td>
</tr>
<tr>
<td>2 Wake effect $\xi_{\text{wake}}$</td>
<td>3.6257 · 10^6</td>
<td>0.21 %</td>
<td>5.7514 · 10^6</td>
</tr>
<tr>
<td>$C_{t}$ curve $\xi_{\text{ct}}$</td>
<td>3.6569 · 10^6</td>
<td>0.21 %</td>
<td>6.0823 · 10^6</td>
</tr>
<tr>
<td>Surface roughness $\xi_{\text{rough}}$</td>
<td>2.7336 · 10^6</td>
<td>0.16 %</td>
<td>3.9914 · 10^6</td>
</tr>
<tr>
<td>3 Power curve $\xi_{\text{power}}$</td>
<td>5.6519 · 10^7</td>
<td>3.29 %</td>
<td>2.6036 · 10^8</td>
</tr>
<tr>
<td>4 Plant performance $\xi_{\text{performance}}$</td>
<td>7.3561 · 10^7</td>
<td>4.28 %</td>
<td>2.862 · 10^8</td>
</tr>
</tbody>
</table>

### 5.2. Levelized costs of electricity

In Figure 14 the results regarding the levelized costs of electricity for all nine influencing uncertainty parameters are shown, according to an equal deviation of 5% (academic test case) and according to the deviations given in Table 4 (realistic test case).

As before, it can be seen that the main impact results from the uncertainty of the wind speed. Another interesting result is that the outliers of the combined and wind speed box plot only spread in the direction of a higher LCOE, which again speaks for the high sensitivity of the wind speed.

Table 7 and Table 8 show the results in terms of the variance-based sensitivity analysis for the academic and realistic test case. Because the LCOE highly depends on the net AEP, the uncertainties show a similar behavior as before. In
the realistic test case, for all three wind farms the sensitivity value is higher than 78 %, while the sensitivities of the power curve and plant performance follow with about 3 to 8 %. Furthermore, the sensitivities for the capital costs and the annual O& M costs are in-between 1.4 and 7.7 %. All remaining uncertainties (wake effect, $C_t$ curve, surface roughness and discount rate) are below 1.2 % and thus have a rather negligible influence on the variance of the model output.
Figure 14: Graphical visualization of the sensitivity analysis with all nine uncertainty parameters wind speed, wake effect, $C_t$ curve, surface roughness, power curve, plant performance, capital costs, O&M costs and discount rate at the same time (simultaneously) compared to single effects of the uncertainty parameters computed with the quasi-Monte Carlo method with 100,000 samples for the Horns Rev 1, DanTysk and Sandbank wind farms.
<table>
<thead>
<tr>
<th>Uncertainty</th>
<th>Horns Rev 1</th>
<th>DanTysk</th>
<th>Sandbank</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>variance</td>
<td>sensitivity</td>
<td>variance</td>
</tr>
<tr>
<td>Wind speed</td>
<td>$\xi_{\text{wind}}$</td>
<td>6.7186 x 10^{-1}</td>
<td>42.13%</td>
</tr>
<tr>
<td>Wake effect</td>
<td>$\xi_{\text{wake}}$</td>
<td>2.6063 x 10^{-2}</td>
<td>1.63%</td>
</tr>
<tr>
<td>$C_v$ curve</td>
<td>$\xi_{C_v}$</td>
<td>2.7456 x 10^{-2}</td>
<td>1.72%</td>
</tr>
<tr>
<td>Surface roughness</td>
<td>$\xi_{\text{rough}}$</td>
<td>2.3825 x 10^{-2}</td>
<td>1.49%</td>
</tr>
<tr>
<td>Power curve</td>
<td>$\xi_{\text{power}}$</td>
<td>3.4543 x 10^{-1}</td>
<td>21.66%</td>
</tr>
<tr>
<td>Plant performance</td>
<td>$\xi_{\text{performance}}$</td>
<td>3.433 x 10^{-1}</td>
<td>21.53%</td>
</tr>
<tr>
<td>Capital costs</td>
<td>$\xi_{\text{capital}}$</td>
<td>4.3827 x 10^{-2}</td>
<td>2.75%</td>
</tr>
<tr>
<td>Annual O&amp;M costs</td>
<td>$\xi_{\text{O&amp;M}}$</td>
<td>2.1092 x 10^{-1}</td>
<td>13.23%</td>
</tr>
<tr>
<td>Discount rate</td>
<td>$\xi_{\text{rate}}$</td>
<td>2.4519 x 10^{-2}</td>
<td>1.54%</td>
</tr>
</tbody>
</table>

Table 7: Results of variances based sensitivity analysis where all uncertainties are fixed to 5% of LCOE calculation with 100,000 samples using the quasi-Monte Carlo Method.

<table>
<thead>
<tr>
<th>Uncertainty</th>
<th>Horns Rev 1</th>
<th>DanTysk</th>
<th>Sandbank</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>variance</td>
<td>sensitivity</td>
<td>variance</td>
</tr>
<tr>
<td>Wind speed</td>
<td>$\xi_{\text{wind}}$</td>
<td>6.3759 x 10^{-1}</td>
<td>90.15%</td>
</tr>
<tr>
<td>Wake effect</td>
<td>$\xi_{\text{wake}}$</td>
<td>8.1103 x 10^{-3}</td>
<td>1.15%</td>
</tr>
<tr>
<td>$C_v$ curve</td>
<td>$\xi_{C_v}$</td>
<td>8.0655 x 10^{-3}</td>
<td>1.14%</td>
</tr>
<tr>
<td>Surface roughness</td>
<td>$\xi_{\text{rough}}$</td>
<td>7.8208 x 10^{-3}</td>
<td>1.11%</td>
</tr>
<tr>
<td>Power curve</td>
<td>$\xi_{\text{power}}$</td>
<td>2.6861 x 10^{-2}</td>
<td>3.8%</td>
</tr>
<tr>
<td>Plant performance</td>
<td>$\xi_{\text{performance}}$</td>
<td>3.2909 x 10^{-2}</td>
<td>4.65%</td>
</tr>
<tr>
<td>Capital costs</td>
<td>$\xi_{\text{capital}}$</td>
<td>1.0469 x 10^{-2}</td>
<td>1.48%</td>
</tr>
<tr>
<td>Annual O&amp;M costs</td>
<td>$\xi_{\text{O&amp;M}}$</td>
<td>3.2609 x 10^{-2}</td>
<td>4.61%</td>
</tr>
<tr>
<td>Discount rate</td>
<td>$\xi_{\text{rate}}$</td>
<td>7.9749 x 10^{-3}</td>
<td>1.13%</td>
</tr>
</tbody>
</table>

Table 8: Results of variances based sensitivity analysis of LCOE calculation with 100,000 samples using the quasi-Monte Carlo Method.
5.3. Discussion of the results

For the three existing wind farms Horns Rev 1, DanTysk, and Sandbank a sensitivity analysis with regard to the two different objective functions net AEP and LCOE was performed. For the deviation of the uncertainty inflicted parameters we chose two different settings (academic and realistic test case).

The wind speed has the largest impact with a sensitivity of higher than 78 %. This result shows that the assessment of the wind resource is of very high importance. This effect can also be seen in the convergence plot of the UQ methods in Figure 9, as quasi-Monte Carlo and Stochastic Collocation show faster convergence than expected.

The uncertain parameters describing the wake effect, $C_t$ curve, surface roughness and discount rate have a sensitivity below 1.2 %. In further investigations, these values should not be considered anymore to save computation time. The other four sensitivities for power curve, plant performance, capital costs and annual O&M costs are in-between 1.4 and 9 % and could therefore be considered in further investigations.

As UQ method, quasi-Monte Carlo and the Stochastic Collocation show the fastest convergence rate. For the ease of implementation and a fine-grained error control, we recommend the quasi-Monte Carlo method.

6. Conclusion

Within this work we investigated the sensitivity of certain input parameters for the estimated economics of offshore wind farms. We developed a model describing the wind, wake, and power generation of an offshore wind farm which delivers an output of two economic goal functions, i.e. the net AEP, and the LCOE. These goal functions are usually used for site evaluation and construction feasibility. We extended the deterministic model by introducing up to nine uncertain model parameters, e.g. uncertain wind speed and wake effects. Due to nature of the resulting stochastic model solution methods from the field
of Uncertainty Quantification are needed. Several methods were compared in terms of convergence rate, overhead, and ease of implementation.

As a result of this investigation, the quasi-Monte Carlo method turned out to be used for the propagation of the uncertainties for the underlying problem of a simulation of a wind farm, as this method offers fast convergence rates, has low implementation effort and allow for a good error control.

In a case study we performed a sensitivity analysis on three existing wind farms, i.e. Horns Rev 1, DanTysk, and Sandbank. We investigated the influence of our up to nine uncertainty inflicted parameters on two economic quantities.

Our results show for all wind farms and for all objective functions, that the wind speed has the largest influence on the variance of the model output. Furthermore, four more parameters could be considered for further sensitivity analyses, while the remaining four parameters are negligible.

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References


