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Optimale Kabelführung in Offshore-Windparks
Shortest cable routing in offshore wind farms

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1. Introduction

1.1. Motivation

Over the last decades the concept of sustainability became increasingly important. Especially in the energy sector a large number of new technologies are based on this notion. One large branch of renewable energies is wind power. The kinetic power of wind is used to drive a turbine which is connected to a mechanical generator for electricity. There are onshore and offshore facilities. Offshore wind facilities do have the advantage of a constant strong wind, but also the disadvantage of larger installation and maintenance costs. To make the farms economically viable those costs need to be kept at a minimum. Therefore offshore wind turbines are often installed in large wind farms with the aim of only needing one cable connecting the whole park to the shore.

The costs for the cables, their installation and the power losses make over 47 % of the total costs for an offshore wind farm, as shown in Figure 1.

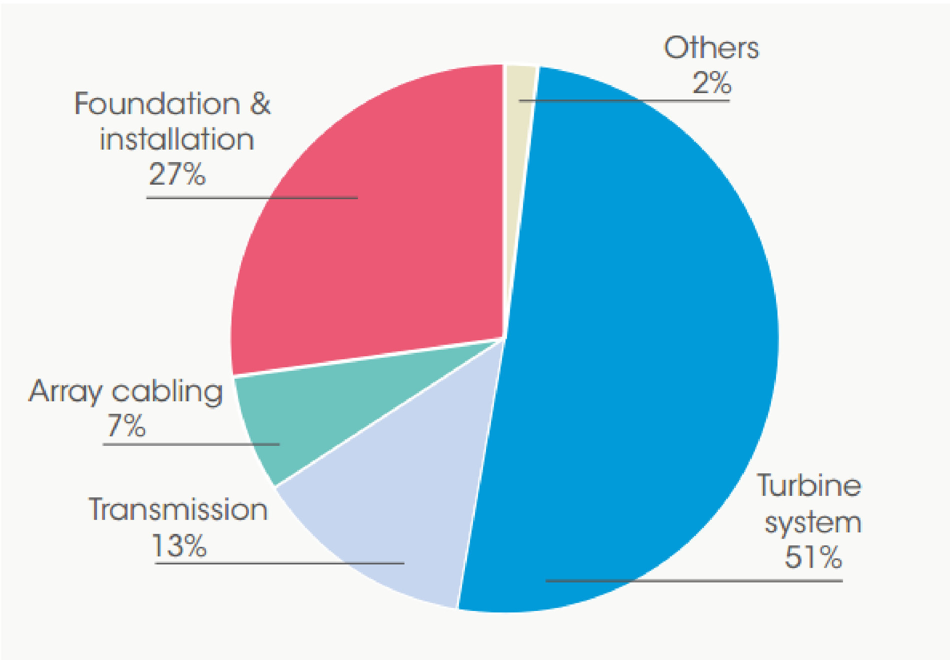

Figure 1: Offshore cost distribution cf. [9]

So far the cabling costs are not considered int the layout optimisation process. Until now even large companies running offshore farms still search the layout for the cabling manually. The goal of this thesis is to find an algorithm, that computes the optimal costs for installation, cables and power losses depending on the locations of the turbines. If these costs are known, they directly can be included into the layout optimisation process. Therefore a fast working algorithm is searched.

1.2. Cabling of offshore wind farms

This section explains the cabling of an offshore wind farm. Figure 2 shows the basic cabling of an offshore wind farm. Commonly wind parks consist of a set of wind turbines placed on a given sea area. The turbines are connected with each other by the infield power collection. One or several shore connection cables transmit the gained energy from the farm collection point to the shore, where it is fed in the public grid by an integrated grid connection point.

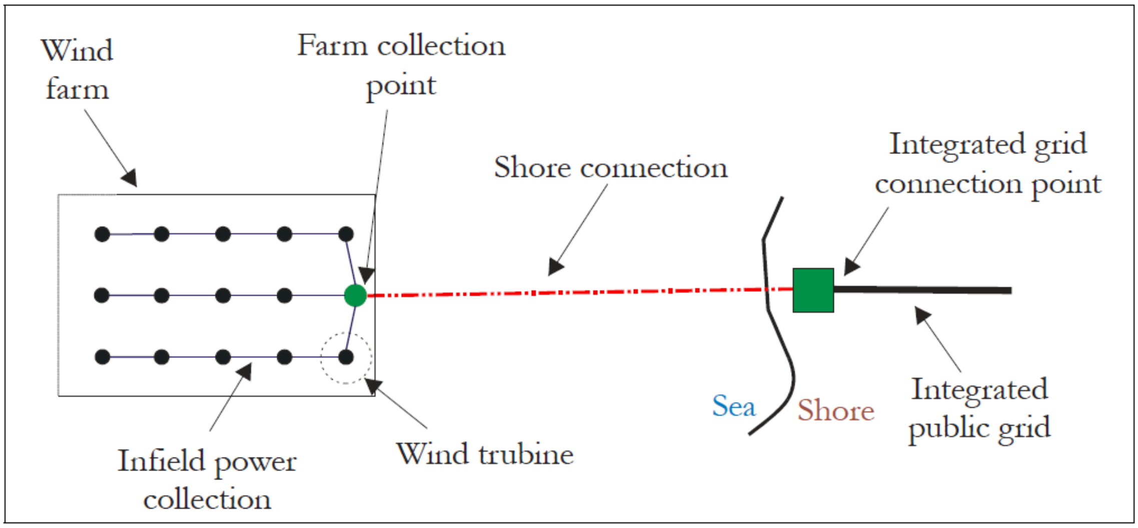

Figure 2: Cable connection in an offshore windfarm cf. [19]

The optimal choice of the cable types for the infield power collection and the shore connection was already investigated in [19]. Since overhead transmissions are expected to have much higher installation costs because of the need for several offshore structures, they are not considered in [19] and also not in this work. Sub-sea power cables are commonly used to transport electrical energy across the sea.
There are two main operation modes for the shore connection: alternating current (AC) and direct current (DC). A turbine produces alternating current. Therefore it makes sense to use an AC connection for the infield power collection. Typically in medium voltage. AC is easier to transform to other voltages. Therefore the power is also onshore transmitted by AC, compare to Figure 3.

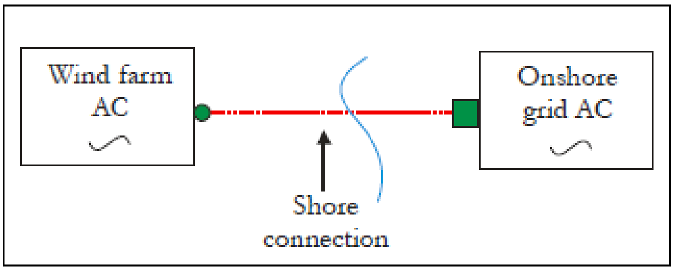

Figure 3: Current type onshore and infield, cf.[19]

To avoid the need of a transformer the obvious solution would be to connect the farm collection point and the shore with an AC connection. But the transportation capacity of medium volatage cables is limited and often more than one cable would be necessary to transport the energy. The other limiting factor of a medium voltage cable is the ohmic resistance of the cable itself. The exact calculation is described in section 6.2.

The maximal economically reasonable length for an AC connection under water is approximately 100 km considering the loss and the cost of the transformers, cf. [20]. This is shown in the line graph in Figure 4. It shows the dependence of the costs including losses on the length of an AC or DC connection. The costs are given relatively to the cable costs of an DC connection.

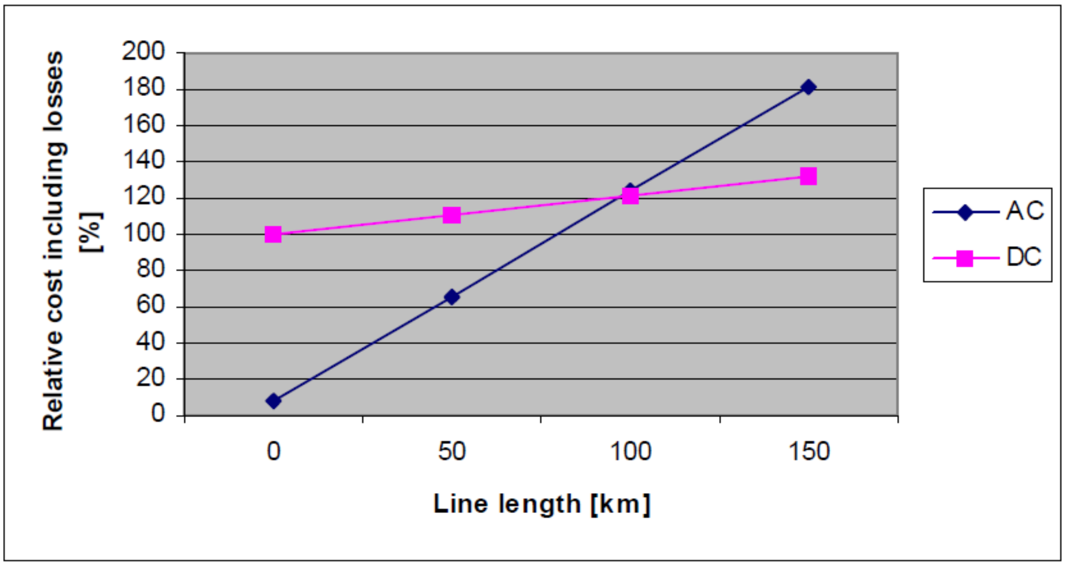

Figure 4: Relative cost including losses, cable costs and in case of DC transformer, cf.[20]

An example of an DC shore connection is the BARD offshore 1 project [21]. The German wind farm is placed 126 km offshore in the North Sea.

The Horns Rev wind farm, which is the test case for the algorithms, is an example for an AC connected wind park. It is placed 14-20 km offshore in the North Sea and connected with AC at 150 kV. The layout of the Horns Rev wind farm is shown in Figure 5. The transformer station also known as substation is in the actual layout placed int the upper right corner.

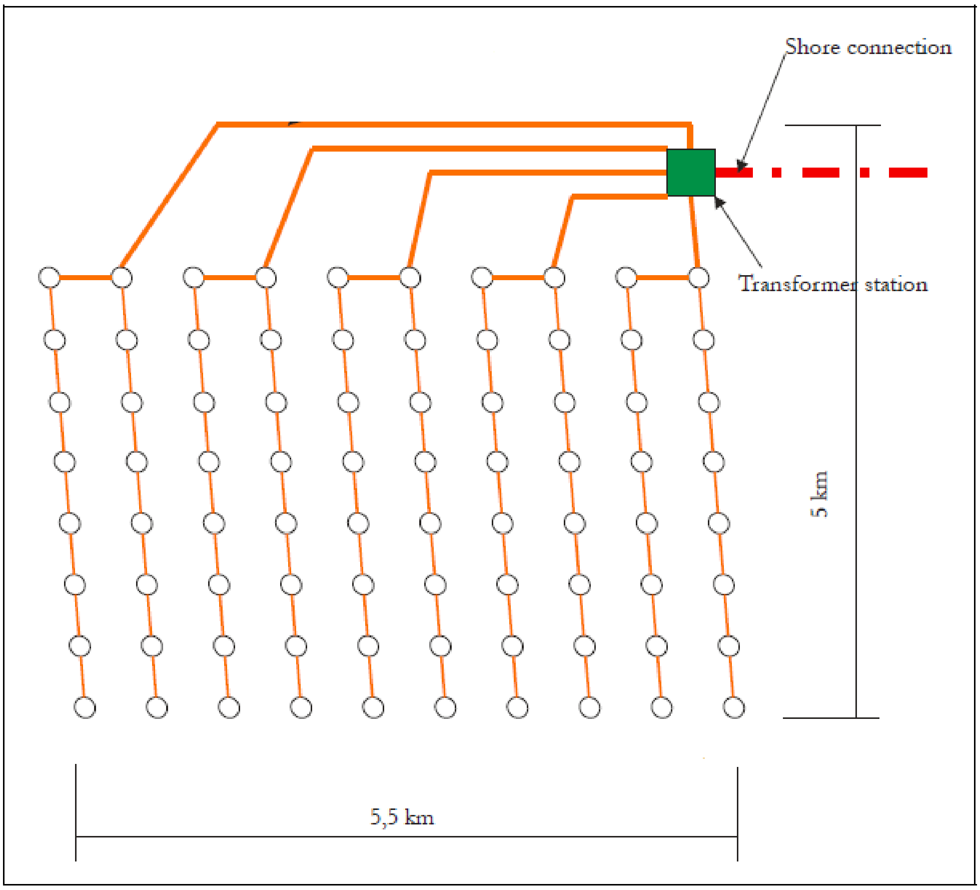

Figure 5: Layout of the Horns Rev wind farm, cf.[19]

1.3. Preliminary works

Scientific works on the optimal cable routing of an offshore wind farm can be divided into two big groups.

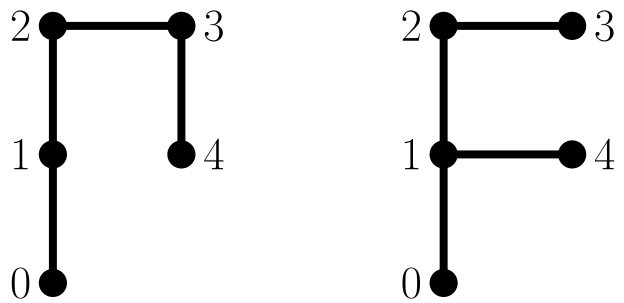

Figure 6: Example for a connection using only strings (left) and with branching (right)

The first group of layouts is using only cable strings to connect the turbines to the substation, e.g. [18], [2]. This means each turbine has a maximum of two cables connected to itself, as shown in the left example in Figure 6. One input, receiving the power of the previous turbine and one output that transfers the power to the next turbine.

The other group allows "branching" of the cables where a turbine is placed. That means one turbine may receive power through more than one cable, as shown in the right example in Figure 6. Here Turbine 1 receives the power of Turbine 2 and 4. This kind of layout is only seldom discussed yet.

But as listed in [11] there are a lot of advantages in allowing branching:

The main advantage of a branched layout is the savings in the total cable length. Since solutions for a string layout form a subset of the solutions for the branched layout, the optimal solution with branching should have at most the cable length as the optimal solution without branching has.

Furthermore, in the branched solution there are less cables with a high transmission capacity needed. In Figure 6 the branched layout needs two cables of capacity 1 and one of capacity 2 and 4, whereas the stringed layout needs one of each capacity type. Therefore the branched layout in general saves money, since cables with a higher transmission capacity are more expensive.

Another advantage of a branched solution over a string solution are the smaller power losses caused by a cable fault within the working layout. If one assumes that in Figure 6 the cable connecting Turbines 1 and 2 has a fault, one loses the power of Turbines 2, 3 and 4 in the string version, but only the power of Turbines 2 and 3 in the branched version of the layout.

We also want to consider the power losses in our algorithm. This aspect is not included in most of the layouts of other works. The main losses are the ohmic losses which are proportional to the square of the current I. If one supposes that the length of every cable is the same, the losses of the branched layout account for only 73.3% of the losses of the stringed layout.

The common reason why branches of the cables in the layout should not be allowed is the higher installation costs of connecting more than two cables to a turbine. But there are also works claiming the opposite, that branching can be done ”without significant additional effort or cost, which opens the possibility of a further reduction of the total required cable length” [12]. More information on this in section 9.

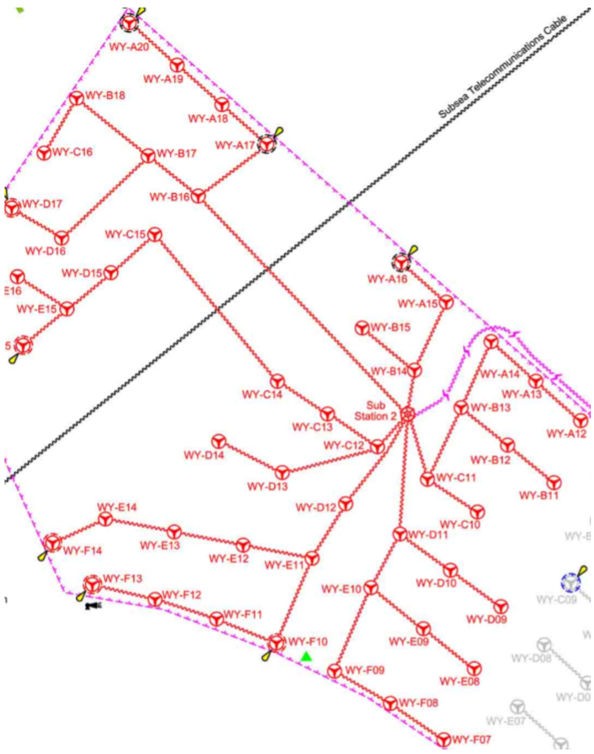

Figure 7: Grid of the offshore wind farm Walney 2, cf. [22]

As an example, the layout of the Walney 1 and 2 offshore wind farms has a branched grid. It is placed 14 km west of the Walney Island in the East Irish Sea and was commissioned in 2012.

By virtue of the above mentioned reasons, the algorithm for the layout of this thesis allows branching of the cables at the locations of the turbines.

1.4. Definition of the problem and approach

First of all, we need to clearly define our problem and the properties of the cabling layout. In this thesis we search a quick working algorithm finding an optimized cable layout for the infield power collection of turbines. Below, the considered characteristics for the cabling layout of an offshore wind farm are described. More information on the properties can be found in the following sections.

- Costs for different cables and their installation are considered and can be varied individually.
- Cables are not allowed to cross.
- The power losses caused by the transmission through different types of cables are considered.
- The irregularities of the seabed are considered since they lead to longer cables and therefore higher costs.
- The placing of the substation is often given by the laws of the country. different parties. The plant operators would prefer to have the substation in the center of the turbines, whereas the grid operator for electricity prefers to have the substation close to the grid. Who is responsible for the actual placing depends on the country. In that event it does not make sense to optimise the location of the substation. Therefore our program allows the user to vary the location of the substation individually.
- The developed program will later be used within an optimization process for wind farm layouts. Therefore the algorithm should be as fast as possible.

In section 8 a few more features are discussed that may be important for the layout but go beyond the scope of this thesis.

The algorithm for the layout is built step by step treating all the properties by using a new algorithm or modifying one of the already implemented ones. The algorithms are implemented using MATLAB.

All algorithms are applied for the Horns Rev wind farm. The original layout of the Horns Rev wind farm is shown on Figure 5 in Section 1. The visualisation is done using MATLAB. If different cable types are used the differences of the cables are indicated using the thickness and colour of the cables in the figure. The brighter and thicker the cable is, the higher its capacity and the thinner and darker a cable, the lower is its capacity.

2. Minimization of the cable length

The costs for the cables and their installation is one of the features that influences the outcome of the layout the most. Submarine cables are very expensive and the higher the transmitted power, the greater the cross section area of the cable needs to be and therefore the higher the costs are.

It is also important to take the installation costs into account [24]. The costs can vary due to different kind of soils depending on the needed time to dig through it. But these detailed costs are often not known in advance. For simplicity, the study assumes that the area of the infield power collection does not contain any different kind of soils. Only the locations where cables can be installed and locations where cables can not be installed is set apart.

2.1. Minimum spanning tree problem

If we only consider the length of the cables, we can trace back the problem to the abstract problem of finding a multilevel capacitated minimum spanning tree. The basic problem of the minimum spanning tree would correspond to the problem of using only one cable type that is able to connect an unlimited number of turbines.

Here the definition and algorithms solving the problem of a minimum spanning tree are given:

Preconditions: Consider a given set of nodes N=$\{N_1, N_2, ..., N_n\}$ and a graph that connects them. From each node there has to be a path to any other node. The weight, in our example the (cable) cost, to connect any node N_i directly to any other node N_j is saved in the edge $e_{i,j}$. If nodes are not directly connected in the graph, the value of their corresponding edge is infinity. Since the cost $e_{i,j}$ equals the cost of $e_{j,i}$ for any $i, j \in \{1,...,n\}$ the graph is undirected.

Objective: A tree is an undirected sub graph in which any two vertices are connected by exactly one path. In the minimum spanning tree problem the objective is to find a minimum weighted tree, connecting all nodes of the given set N.

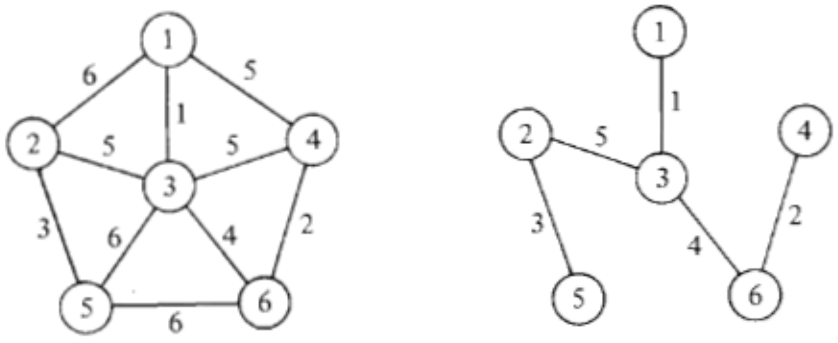

Figure 8: A graph and one of its minimum spanning trees, cf.[23]

Other applications: The problem of a minimum spanning tree also appears in other fields of industry. For example, as described in [1], if oil companies wish to connect a network to carry oil from several stations to a refinery.

Solution / Algorithms: There already exist algorithms that solve the problem in polynomial time FP[1]. Two of the algorithms, Kruskal's and Prim's are described below, compare [5]. Both of them are "Greedy Algorithms", that means in each step they make a locally optimal choice which is not changed in later steps.

- Prim's algorithm:
  - define *in tree nodes* and *in tree edges* as empty sets
  - choose a start node and add it to the set *in tree nodes*
  - find a minimum weighted edge that connects a node which is not yet in the tree with a node of *in tree nodes*, add this edge to *in tree edges* and the node to *in tree nodes.*
  - repeat step above until *in tree nodes* contains all nodes

  Runtime: The used implementation has a runtime of $\mathcal{O}(n^3)$. There are more advanced implementations possible that only need $\mathcal{O}(n^2 + n \cdot log(n))$. But since the used number of nodes in this thesis is about 100 the difference in the runtime is very small.

- Kruskal's algorithm:
  - define *in tree edges* as an empty set
  - take the edge with the minimum weight, which is not already in *in tree edges* and does not produce a *circle* within the tree and add it to the tree
  - repeat step above until *in tree edges* contains n-1 elements

[1] FP=function polynomial time, the set of all function problems that can be solved by a deterministic Turing machine in polynomial time.

Runtime: The scaling of the runtime of Kruskals algorithm is $\mathcal{O}(n \cdot log(n))$, which is even better than the best implementation of Prim's Algorithm.

Prim's Algorithm can be easily modified for the working in different cable types. Therefore already thinking ahead, Prim's algorithm is chosen to be implemented although Kruskal's algorithm has a shorter running time. Kruskal's algorithm does not start at one edge and expands, but builds the spanning tree more randomly. This algorithm is not suitable for a modification which uses different cable types. Therefore Prim's algorithm is the best choice in this situation.

The implementation of Prim's algorithm is tested on the example of the Horns Rev wind farm. The two-dimensional location of the turbines is given in Appendix A. This means the irregularities of the sea bed are not considered and the turbines are supposed to be installed at the same depth under the water, as only the x and the y coordinates are given. For the examples the substation is placed in the center of the turbine field. The position of the center is determined by using the average of all coordinates. The algorithms does not depend on the actual position of the wind turbines, but only on the distances between each pair of turbines. The distance $dist_{i,j}$ from turbine i to turbine j is easily calculated by use of the Pythagorean theorem:

$$dist_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

The resulting layout is shown in Figure 9.

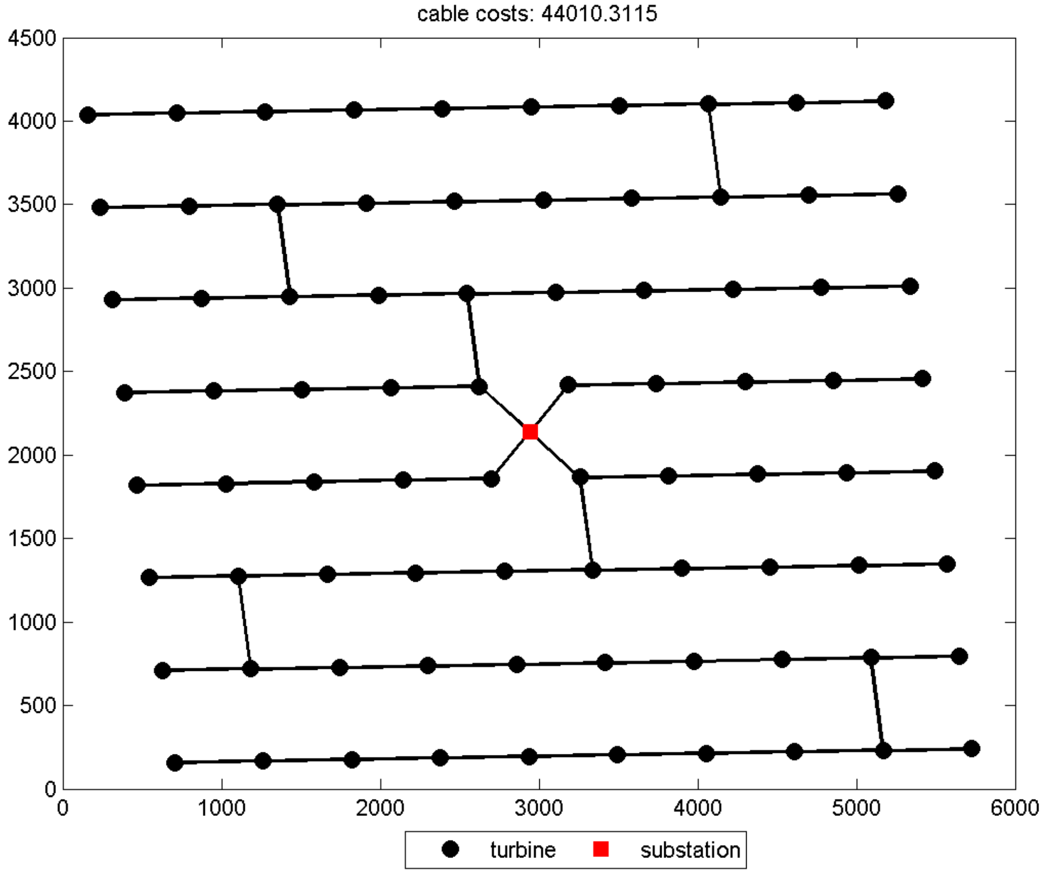

Figure 9: Prim's algorithm applied to the turbines' locations of the Horns Rev wind-farm. The cable costs are calculated with the information that one meter of cable costs one monetary unit.

The distances in x direction are smaller than the ones in y direction. Therefore every turbine is connected to the neighbours in x direction. Certainly these strings of cables connecting turbines need one connection to the next line. Where exactly these strings are connected is not uniquely defined. Here it depends on small deviations of the given data or how the turbines are numbered within the algorithm, if more than one solution is optimal, the first found solution is privileged by the algorithm.

3. Consideration of the current-carrying capacity of cables

Each cable has a current-carrying capacity. Transmitting energy that exceeds the allowed capacity of the cable would cause damage to the cable. Therefore each cable type has a maximum number of wind turbines the cable can connect. For each layout often two or three different cable types are used. Hence there is a maximal number of included turbines in a branch starting from the substation that must not be exceeded.

If more than one cable types with different costs and capacities are considered, the problem corresponds to the mathematical problem of a *Multilevel Capacitated Minimum Spanning Tree*, which is also called *Single-Sink Buy-At-Bulk*:

3.1. Capacitated minimum spanning tree

Preconditions: All preconditions of the basic minimum spanning tree are also valid for the capacitated minimum spanning tree. Additionally, there is a given number a of cable types (only one for the normal capacitated minimum spanning tree problem). Each cable i has a maximum capacity z_i and a costs per unit c_i. Every node N_i has a capacity requirement r_i. Since every turbine is assumed to produce the same amount of power, all nodes have the same capacity requirement r_i=1 except for the substation (N_1), with r_1=0. In other applications the precondition, that the cables satisfy the economies of scale principle is often given. This means: the cost per unit capacity and unit length decreases from small to large cables. This is no precondition of this problem.

Objective: The general objective is the same as in the basic problem but with a higher complexity. This time every node needs to be connected to a substation with cables that have an capacity that must not be exceeded. Furthermore the costs for each cable type are different. All of that complicates the task of finding a solution and optimising the costs to connect every node (turbine) to the central node (substation). In Figure 10 an example of an multilevel capacitated spanning tree is shown.

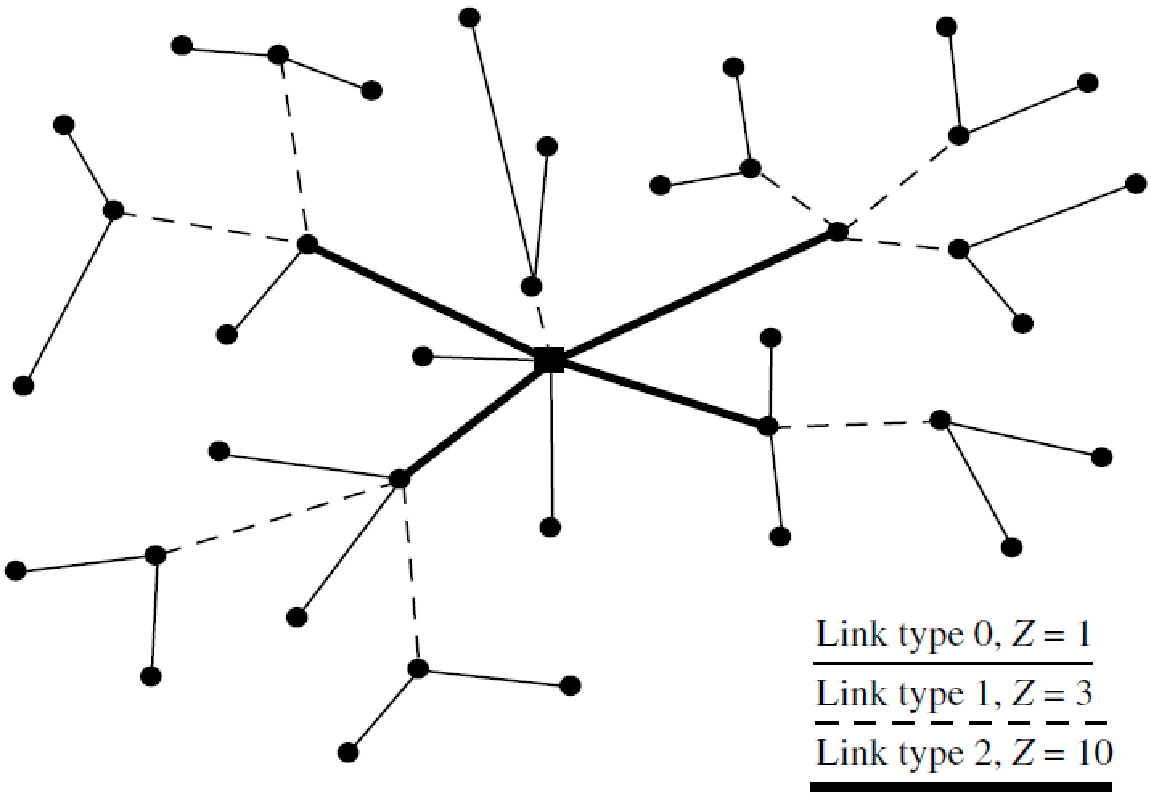

Figure 10: Example for a multilevel capacitated spanning tree connecting the points (nodes) with the square (substation) using three cable types with capacities 1, 3 and 10, cf. [7].

Classification: The capacitated spanning tree problem is shown to be NP-complete[2] [17]. Hence the capacitated minimum spanning tree problem is most probably intractable [17].

Of course one possibility to find an optimized solution for the problem would be to try all spanning tree solutions:

- **Brute-Force-Method:** Cayley's formula determines the number of possibilities of an spanning tree with an given set of n nodes to be n^{n-2}[4]. With an average of 100 turbines and therefore 100 nodes we already would have to check 1e+196 possibilities.

Since this would take too long we have to look for quicker working heuristics giving us solutions that are close to the optimized . One is to use a genetic algorithm as described in [7]. Some more algorithms are describen in section 9.

[2]A problem is NP-complete if any other problem of this complexity class can be leaded back to this problem in polynomial time. Till today there is no algorithm with a polynomial runtime finding the solution of the problems classified as NP-complete.

- **Genetic Algorithm:** First a set of capacitated minimum spanning trees of the given nodes needs to be initialised. This can be done by other faster working algorithms. The next step is it to choose the n best solutions of this set, and build a new set by using connections that can often be found in the best solutions of the previous set. This is done until the termination requirement is fulfilled. For 100 nodes it has an average error of 5.19% and a runtime of over 1000 seconds. The given runtime is taken of an implementation in Visual C++. All runs are conducted on a dual-processor Pentium III PC running Windows 2000, 1 GHz clock speed, with 512 MB RAM.

This still takes too long. We therefore first modify our quick working Prim's algorithm:

3.2. Modified Prim's algorithm

Modified Prim's algorithm: It basically works as Prim's algorithm described above. However, in every step we want to add a new node to the already connected tree. Instead of using the normal costs of the edges to determine which node should be connected next, we calculate the costs with the consideration that we might need to change one of the cables to a thicker one and add the extra costs. If the required capacity exceeds the capacity of the biggest cable, a cable with infinity costs is offered to the algorithm. Since costs of infinity do not improve the latest cost, layouts that require cables exceeding the available possibilities are sorted out.

As an example the modified Prim's algorithm is applied to the two dimensional data of the Horns Rev wind farm. Three different cable types that allow the connection of 6, 12 and 24 turbines with costs of 1, 2 and 3 monetary units per meter are used. We assume the cost of the installation of the cable to be 1 unit per meter. The result can be seen in Figure 11.

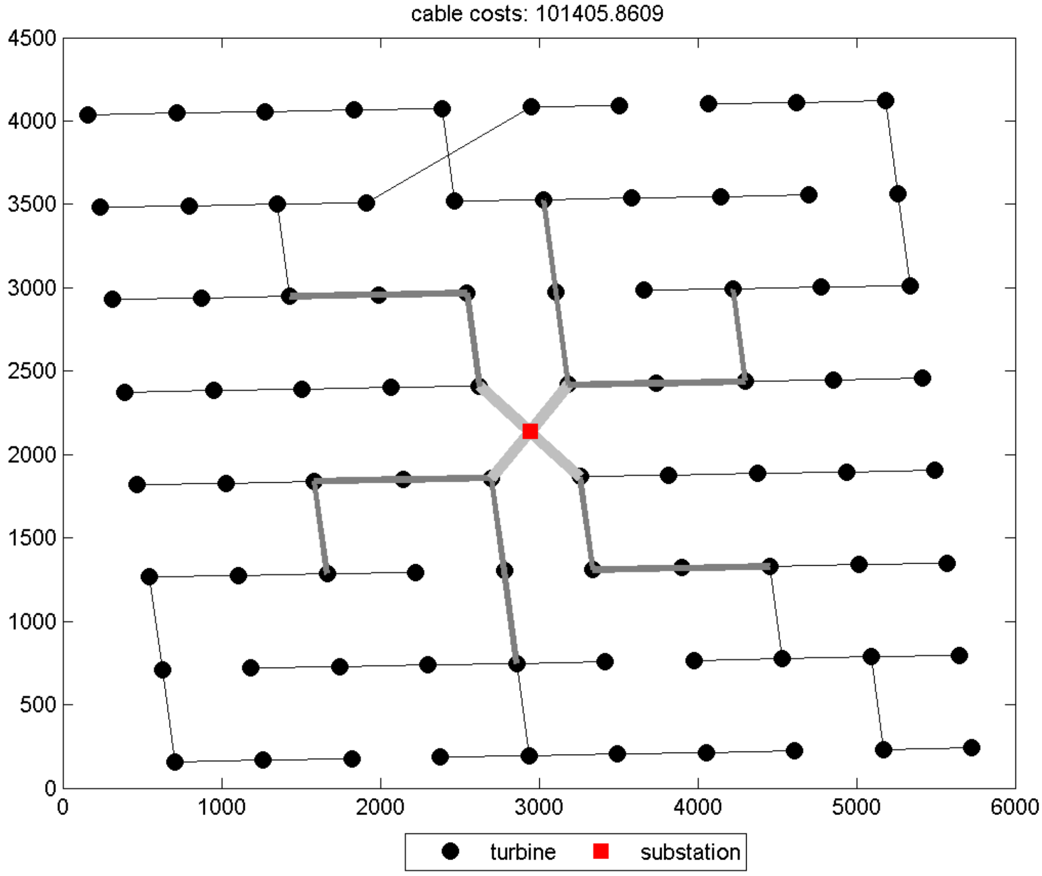

Figure 11: Modified Prime's algorithm applied to the turbine positions of the Horns Rev windfarm in a flat seabed

The first thing to be noticed is the crossing of the cables in the upper left corner. This is not due the crossing itself, but because of the obvious suboptimal cabling the algorithm returns at this point. It is better to connect the two upper left turbines to the left branch and the two center ones that causes the crossings to the right branch.

This sub-optimisation is caused by the fact that modified Prim's algorithm connects the turbines that are closer to the substation first. At the end the two upper middle turbines remain. The branch, to which it would be shortest to connect them to, is, due to the maximum capacity of the cables, not able to take them up. Therefore they must be connected to the branch further away to the left.
The better solution is shown in figure 12.

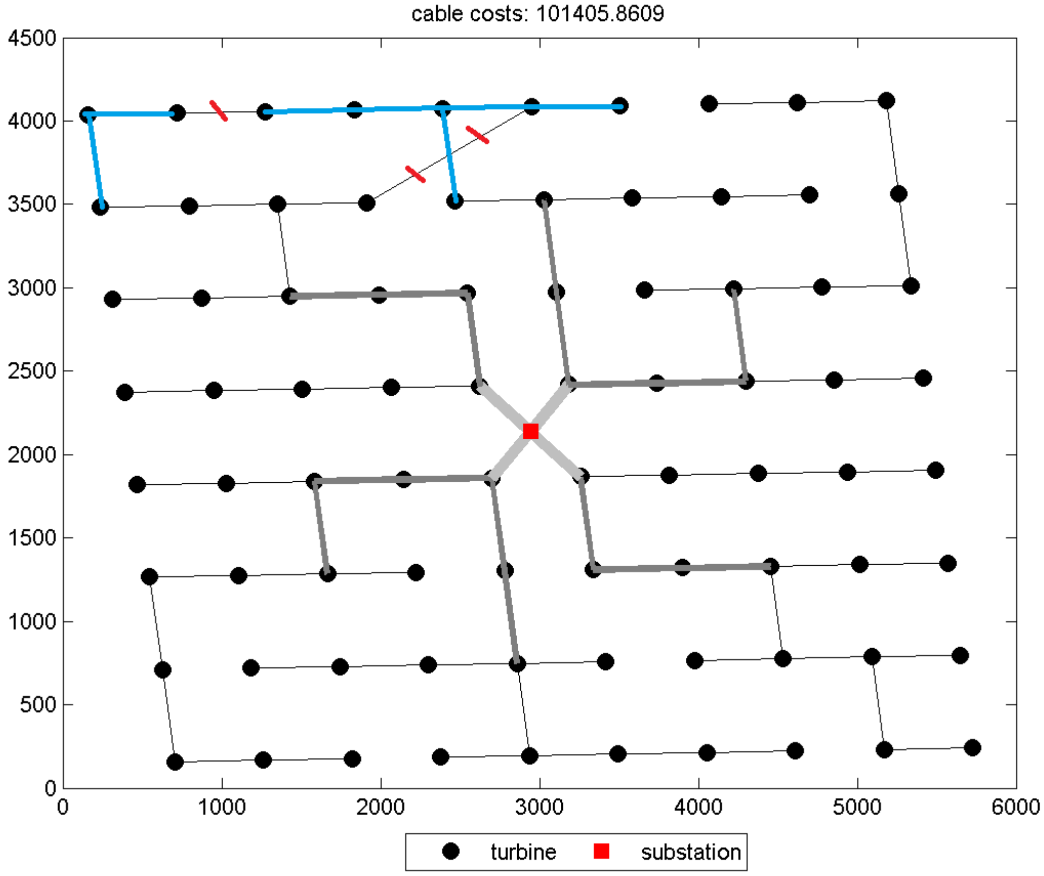

Figure 12: Modified Prim's algorithm applied to the turbine positions of the Horns Rev wind farm in a flat seabed with correction. The improved cabling is marked in blue. The red score marks the cables that are wrongly placed.

To avoid these suboptimal solutions, a new algorithm needs to be implemented, in which the further away turbines are handled first.

3.3. Esau-Williams' Algorithm Version 1

The Esau-Williams Algorithm is predicated on handling distant turbines first. With this approach, the algorithm has a better performance than other algorithms solving this problem.

Therefore this algorithm is the base of the later implemented algorithms. The actual original algorithm was only constructed to solve the problem of a capacitated minimum spanning tree. But it can easily be updated to the multilevel version:

- The first step is to directly connect every node to the substation.
- For all nodes n_j that are directly connected to the substation and for all nodes n_i

that are not connected to the substation via node n_j, calculate the *regret costs*. These are the costs that can be saved by connecting n_j to n_i instead of direct connecting of n_j to the substation. If a cable of higher capacity is needed take the extra costs also into account.

- If the highest *regret costs* are positive, change the connection: connect n_j to n_i and delete the connection from n_j to the substation.
- Repeat the two steps above until the highest regret costs are negative.

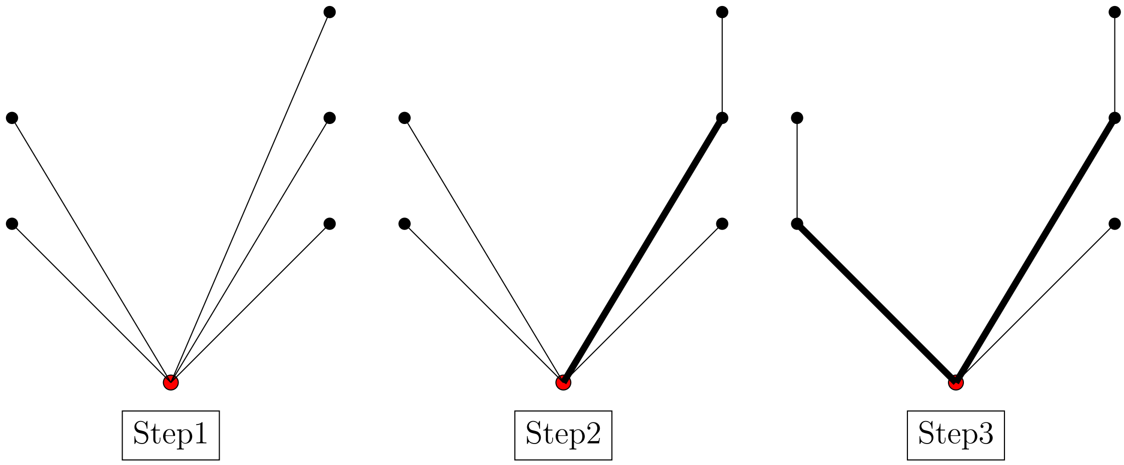

Figure 13: Example of the three steps constructing an capacitated minimum spanning tree with Esau-Williams' Algorithm using cables with capacities 1 and 2.

As described in [10] this algorithm is the most popular and efficient for the capacitated minimum spanning tree problem. It has a runtime of $\mathcal{O}(n^2 \cdot log(n))$. Therefore this algorithm is chosen to be implemented.

In order to have a direct comparison, the same locations of the turbines and the same types and cost of the cables as in the previous example are used to test the Esau-Williams' algorithm. The result can be seen in Figure 14. The cable lengths are kept short and the thicker, more expensive cables are seldom used.

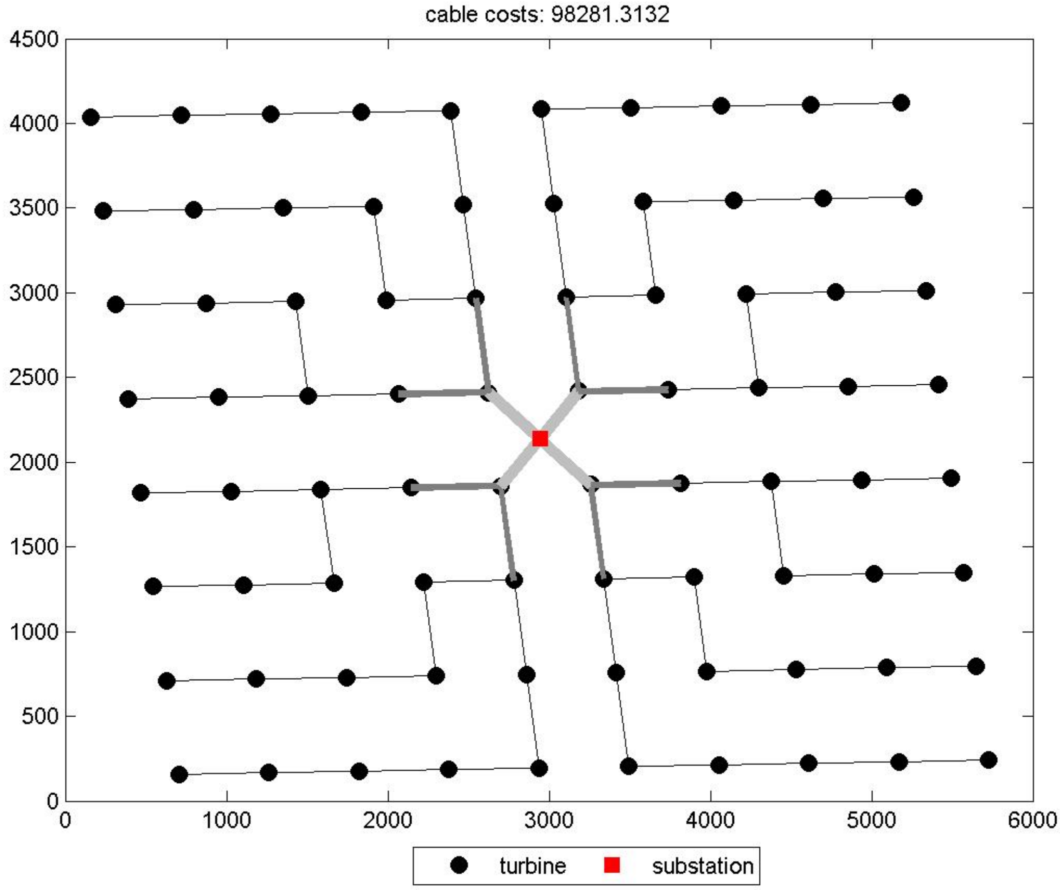

Figure 14: Esau-William's algorithm V1 applied to the turbine positions of the Horns Rev wind farm in a flat seabed.

The cable cost is reduced by more than 3%, from 101,405 monetary units to 98,281 monetary units. The runtime for this example using MATLAB is 3.6 seconds.
The before obvious sub-optimal cabling is avoided. But it can happen that crossings appear, as we will see in Figure 16.

4. Avoid crossings of the cables

The cables need to be buried approximately one meter under the surface. If cables cross each other where no turbine is placed, one cable must be laid below the other one. This leads to higher installation and maintenance costs. That is why cables are not allowed to cross each other. Therefore one has to determine the crucial steps in the algorithm leading to solutions with crossed cables and modify them in order to prevent such behaviour.

4.1. Capacitated minimum spanning and crossing of edges

It is clear that a normal minimum spanning tree in which the weights are defined by the euclidean distances does not include edge crossings.

Therefore it is interesting to know if an euclidean capacitated minimum spanning tree can include crossings of cables. Figure 15 shows that capacitated minimum spanning trees do not have to be crossing free. Four turbines need to be connected to an substation with only using cables of the capacity two. The cost of the cable is one monetary per unit length. Two options for the layout are shown.

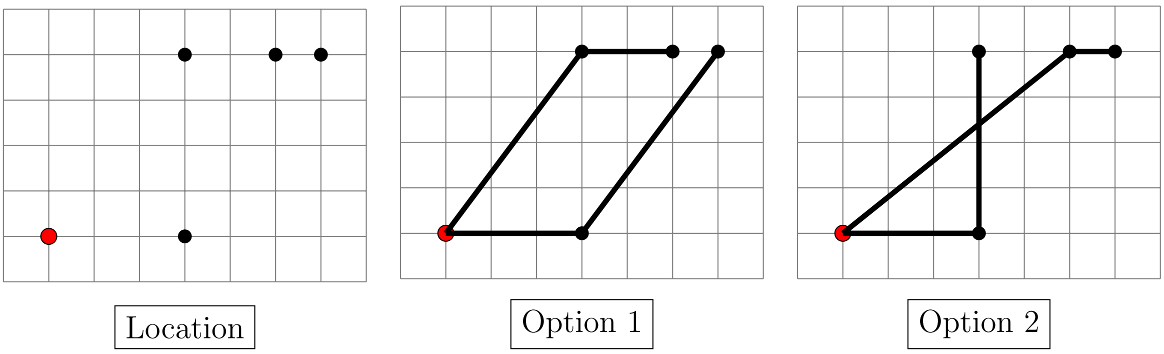

Figure 15: Manageable example for a capacitated minimum spanning tree including a crossing

Any other thinkable option would clearly be more expensive than at least one of those. The costs for option one can be calculated by

$$costs_{option1} = 3 + \sqrt{3^2 + 4^2} + \sqrt{3^2 + 4^2} + 2 = 15 \quad \text{[monetary units]}$$

and the costs for option two by

$$costs_{option2} = 3 + 4 + \sqrt{5^2 + 4^2} + 1 \approx 14.4 \quad \text{[monetary units]}$$

Therefore capacitated minimum spanning trees do not have to be crossing-free.

Also Esau-Williams' algorithm V1 produces layouts including cable crossings. An example is shown in Figure 16, in which only one cable type is used with a capacity of 12 turbines and a price of 1 monetary unit per meter.

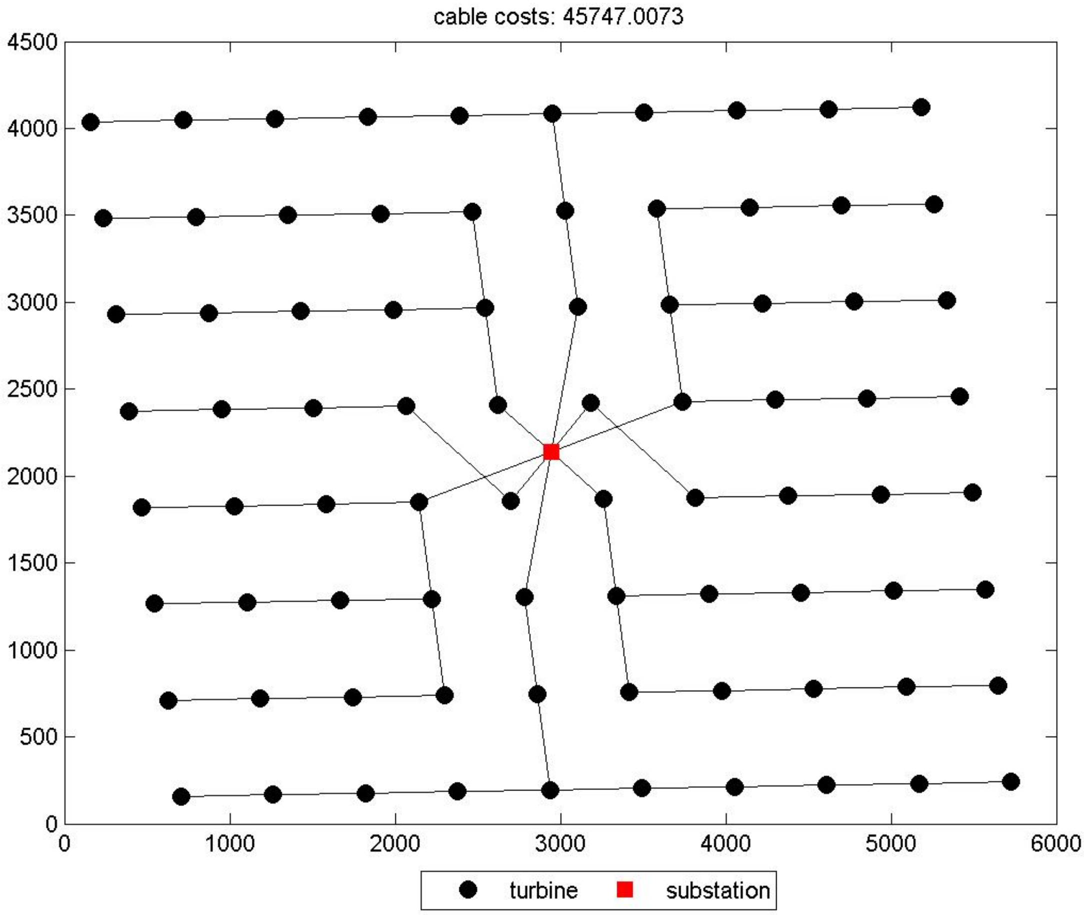

Figure 16: Esau Williams' algorithm applied to the turbine positions of the Horns Rev wind farm in a flat seabed.

4.2. Esau-Williams Algorithm Version 2

To eliminate those crossings we need to update our algorithm to *Esau-Williams' Algorithm Version 2*: Every time a new connection is tested, it is also tested if this connection would cross with any other. If this is the case, the connection is directly suspended from the set of the optional new connections.

With the initial conditions as before a new layout is obtained, shown in Figure 17.

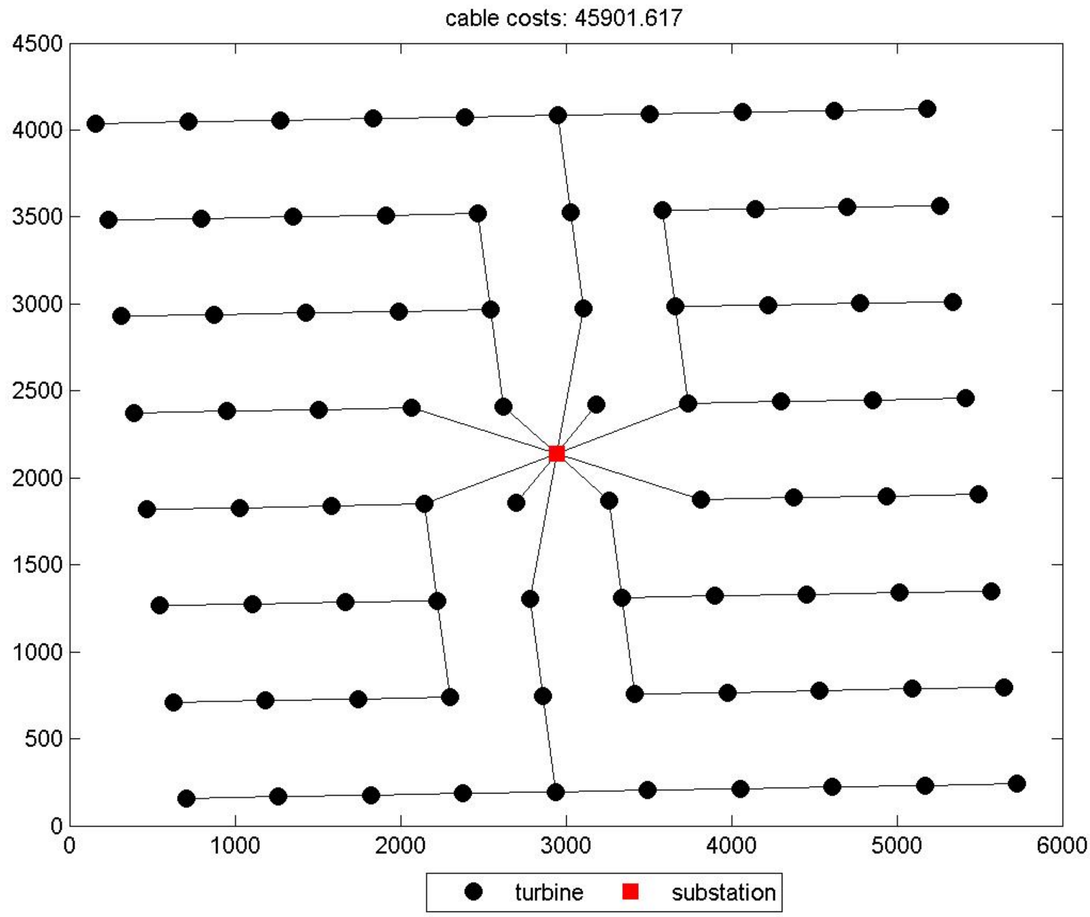

Figure 17: Esau-Williams' avoiding crossings algorithm applied to the turbine positions of the Horns Rev wind farm in a flat seabed.

The method of searching for crossings is time-consuming. A pairwise comparison needs to be done. First of all one needs to determine a linear function defined with the location of the turbines one wants to connect. Next, one has to check if any other function graph defined by two directly connected turbines intersects the function graph. If the intersection is within the area where the functions are actual cables, one needs to exclude the connection, otherwise it can be used. In this example the computation time increases from around 4 seconds to 45 seconds.

This violates the key objective to optimize the performance of the computation. We need to exclude the possibility of cable crossing more efficiently. It is easy to determine whether the options in Figure 18 lead to cable crossing or not.

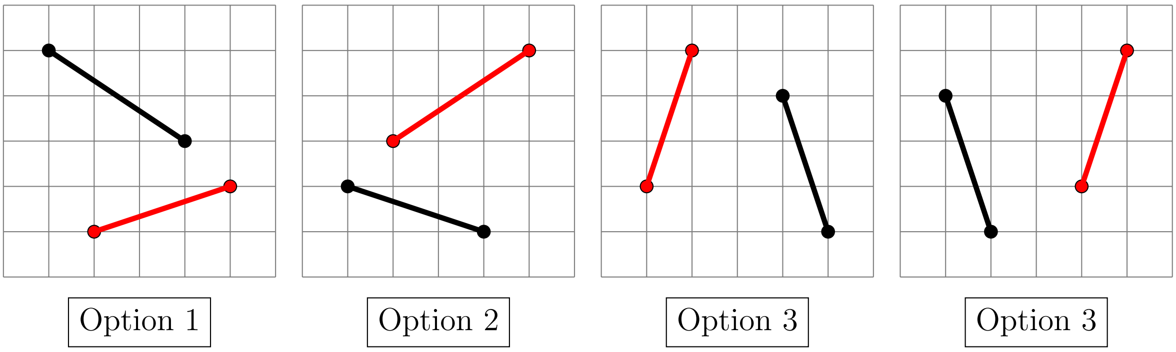

Figure 18: Different options for which it is easy to determine whether cable crossing occurs or not.

We want to check if the red cable would cause an crossing with the black cable.

The cables do not cross if:

- Option 1: Both y coordinates of the turbines defining the black cable are greater than the greatest y coordinate of the red turbines.
- Option 2: Both y coordinates of the turbines defining the black cable are smaller than the smallest y coordinate of the red turbines.
- Option 3: Both x coordinates of the turbines defining the black cable are greater than the greatest x coordinate of the red turbines.
- Option 4: Both y coordinates of the turbines defining the black cable are smaller than the smallest x coordinate of the red turbines.

Implementing if-statements before the complicated method to exclude the obvious cases improves the runtime about 87%, from 45 seconds to 6 seconds.
Another second can be gained by checking for options that definitely causes crossings. They are shown in figure 19.

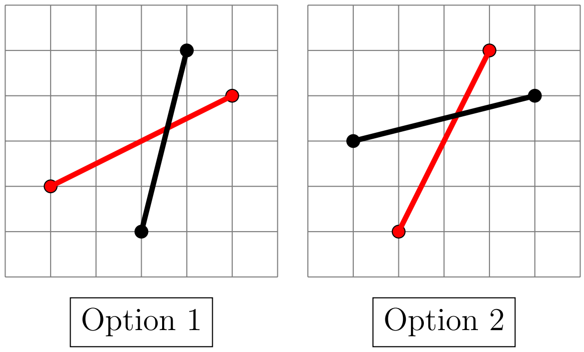

Figure 19: Options for which you can quickly determine that cables do cross each other.

The cable do cross if:

- Option 1: One y coordinate of the turbines defining the black cable is smaller than the smallest y coordinate and the other one is greater than the greatest y coordinate. Additionally the values of both x coordinates of the black turbines are between the values of the x coordinates.
- Option 2: One x coordinate of the turbines defining the black cable is smaller than the smallest x coordinate and the other one is greater than the greatest x coordinate. Additionally the values of both y coordinates of the black turbines are between the values of the y coordinates of the red turbines.

Altogether the avoidance of cable crossings leads to a runtime of 5 seconds and 0,3% higher costs but avoids the described disadvantages explained.

5. Usage of realistic capacities and costs

Before the power losses can be considered within the algorithm, realistic costs for the cables are needed. In order to be able to consider the power losses in the algorithm reasonable information on the cable costs is needed . Otherwise one is not able to compare the costs for the lost power and the cable costs.

5.1. Cable costs

In Table 1 a few different cable types with their costs per meter are listed. The used cable is called ”18/30(36)kV IEC60502-2 Copper Conductor Design With XLPE Insulation Subsea Composite Cables from JDR Cables Systems Ltd”.

Description	Price per m
3 x 150 mm^2	131 €
3 x 185 mm^2	166 €
3 x 240 mm^2	173 €
3 x 300 mm^2	198 €
3 x 400 mm^2	234 €
3 x 500 mm^2	270 €
3 x 630 mm^2	400 €

Table 1: Costs of different cable types, cf. [24]

The installation costs are initiated with 550 € per meter [24].

5.2. Calculations on the capacity of the cables

In Horns Rev wind farm Vestas V80 turbines (Vestas Wind Systems A/S, Randers, Denmark), which are 2 MW pitch controlled, variable speed wind turbines with a 80 m diameter and a 70 m hub height are used. [8]. The data of the submarine power cables of Type (F)2XS2Y>c<RAA in different sizes is used. They allow voltages up to 36 kV [15]. With this information and the formula

$$I = \frac{P}{U} = \frac{2 \cdot 10^6}{36 \cdot 10^3} \approx 55,56 \text{ [A]}$$

the electric current every turbine produces can be calculated. It is approximately $I = 55.56A$. It is important to use the maximal power a turbine produces, since a current exceeding the capacity causes damages on the cable.

The current and the information on the maximum allowed current per cable is used to calculate how many turbines a cable can connect. The results and the numerical value of the maximum allowed electric current per cable are given in table 2.

Description	Current rating (A)	Max. turbines
3 x 150 mm^2	384	6
3 x 185 mm^2	430	7
3 x 240 mm^2	490	8
3x 300 mm^2	543	9
3 x 400 mm^2	600	10
3 x 500 mm^2	659	11
3 x 630 mm^2	721	12

Table 2: Data on XLPE insulated cables at 36 kV, cf. [16]

The calculations for the power losses are implemented, such that the user only needs to put in the raw data as it is shown in Figure 20.

```
%Variables user can change:

    filename='positionshornsRev1flat.dat';
    Voltage=36000;                                      %in volt
    Max_power_per_turbine= 2.0*10^6;                    %in watt
    Max_current_per_phase=[384,543,721];                %in ampere
    Cable_costs_per_meter=[131,198,400];                %in euro
    Installation_costs_per_meter=[550,550,550];         %in euro

%end of variables user can change
```

Figure 20: Necessary input for the modified Esau-Williams' algorithm V2

The x and y positions of the turbines are loaded from a file, see firs line in Figure 20. Next, the voltage, the maximum power a turbine produces and the maximum allowed current in each type of cable need to be given. Additionally the costs of the used cables per meter and the installation costs should be given.
These inputs lead to the cable layout shown on Figure 21.

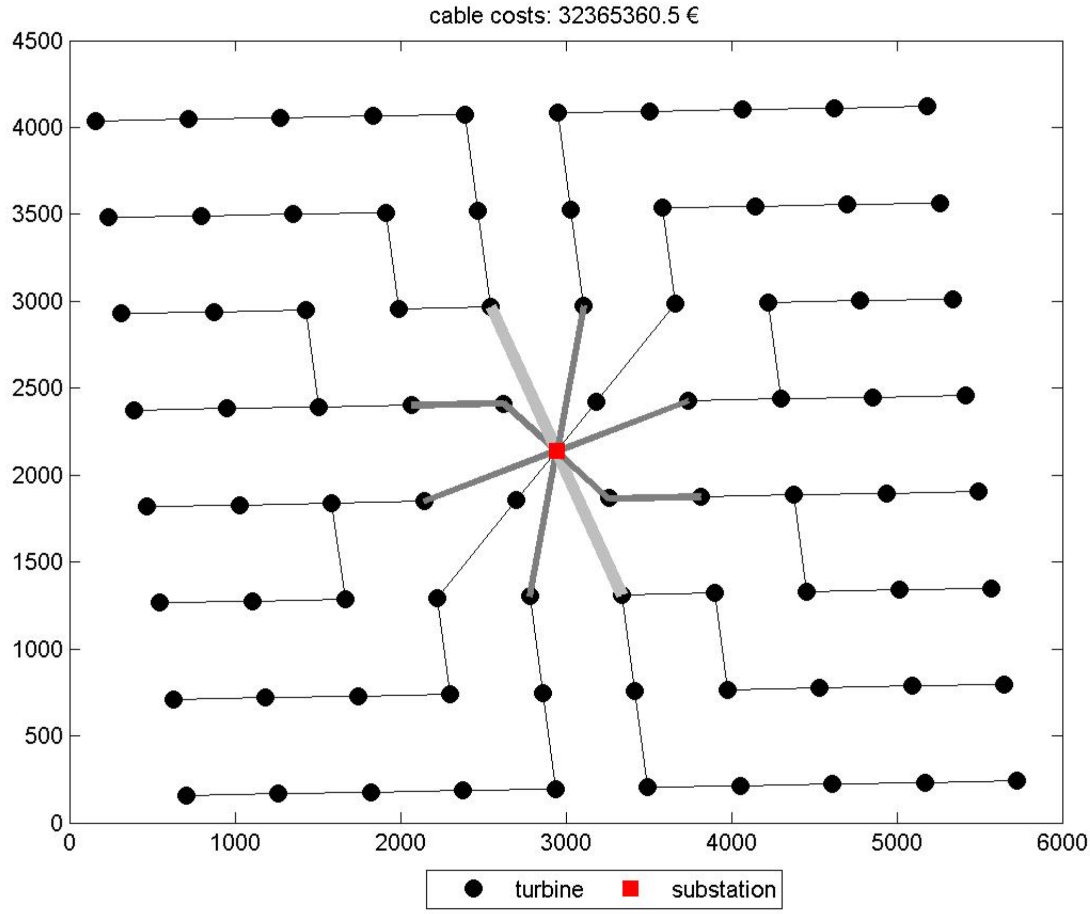

Figure 21: Layout constructed with modified Esau-Williams' algorithm v2

The overall costs for the cables and their installation is shown on the top of the figure. Now, that installation and cable costs are determined, next step is it to take into account the costs for the power losses.

6. Including the power losses

To keep the power losses as low as possible, it would be best, to connect each turbine directly to the substation with a high capacitative cable. This competes with the layout concentrated on the cable costs. To balance these two points we transform the power losses into costs. First the calculation of the power costs needs to be described and then the transformation of the losses into costs will be indicated.

6.1. Power losses

As written in [19], we have two kinds of causes for power losses. The first one is the ohmic loss. Since AC is used for the infield power collection we need to calculate with the AC resistance R_{ac} given in Ω/m. The ohmic losses P_Ω in W/m is calculated by the formula:

$$P_\Omega = R_{ac} I^2.$$

The second one is the dielectric loss W_d, mainly arising when power it is transmitted with AC. It depend on the cable capacity c given in F/m, on frequency f given in 1/s and on the insulation loss factor $tan(\delta)$:

$$W_d = 2\pi f C U^2 tan(\delta).$$

There also is a third cause for losses: The dielectric losses generated in the steel framework of the cable. It surrounds the cable to protect it against damages. There are no formulas of numerical values available on this loss. But [24] claimed that it makes less than 10 % of the overall losses. Therefore they are not taken into account in this work.

6.1.1. Simplification of the calculations for the power losses

To save runtime it is important to know how precisely we have to calculate the costs for the power losses.

First the power losses are calculated without any simplifications. Data transferred from [15] for (F)2XS(FL)2Y>c<RAA 18/30(36) kV cables is used. More data on submarine cables can be found in [13], [19](page 992 to 1000) and [15].

The distance between two turbines is approximately 500 m. The conductor temperature of subsea cables is approximately $90°C$. We take the numerical value of different sized XLPE insulated distribution cables, all working on AC current at 36 kV with a frequency of 50 Hz. The data of the cables is given in Table 3.

Cable type	Conductor size	Maximal current	Capacity	AC resistance	sin(δ)
1	3x150 mm^2	384 A	0,22 μF/km	0,16 Ω/km	0.0004
2	3x185 mm^2	430 A	0,23 μF/km	0,13 Ω/km	0.0004
3	3x240 mm^2	490 A	0,26 μF/km	0,10 Ω/km	0.0004
4	3x300 mm^2	543 A	0,27 μF/km	0,08 Ω/km	0.0004
5	3x600 mm^2	543 A	0,30 μF/km	0,06 Ω/km	0.0004
6	3x659 mm^2	600 A	0,33 μF/km	0,05 Ω/km	0.0004
7	3x630 mm^2	721 A	0,37 μF/km	0,04 Ω/km	0.0004

Table 3: Cable data

To estimate the amount of the losses an example in which up to twelve turbines are connected in a line using cable type 1, 4 and 7 is calculated. The results are shown in Table 4. "Sent power" is the power that has to be transmitted through the cable. "Ohmic" and "Dielectric losses" are the losses occurring in the currently chosen cable. "Overall losses" are the total cumulative power losses due to both ohmic and dielectric losses in the individual cables. The number of connected turbines is given in the row "Connected turbines".

Connected turbines	1	2	3	4	5	6
Sent power [MW]	2.000	4.000	5.999	7.996	9.993	11.986
Ohmic losses [W]	247	986	2221	3947	6164	8869
Dielectric losses [W]	18	18	18	18	18	18
Overall losses [W]	265	1270	3509	7475	13656	22543

Connected turbines	7	8	9	10	11	12
Sent power [MW]	13.997	15.971	17.964	19.954	21.947	23.940
Ohmic losses [W]	5954	7775	9835	6298	7615	9066
Dielectric losses [W]	22	22	22	30	30	30
Overall losses [W]	28519	36316	46173	52501	60151	69246

Table 4: Example of power losses

The total loss to connect 12 turbines is 69246 W. This is about 0.3 % of the originally sent power. In this first example calculation in every step connecting a new turbine, the by the losses traduced arriving power is taken to calculate the losses to the next turbine.

In the next example the calculations are simplified. Now only the originally sent power is taken into account and the former losses are ignored. This simplification leads to the results shown in table 5.

Connected turbines	1	2	3	4	5	6
Sent power [MW]	2.00	4.00	6.00	8.00	10.00	12.00
Ohmic losses [W]	247	988	2222	3951	6173	8889
Dielectric losses [W]	18	18	18	18	18	18
Overall losses [W]	265	1270	3511	7479	13670	22577
Connected turbines	7	8	9	10	11	12
Sent power [MW]	14.00	16.00	18.00	20.00	22.00	24.00
Ohmic losses [W]	5974	7803	9875	6327	7656	9111
Dielectric losses [W]	22	22	22	30	30	30
Overall losses [W]	28572	36397	46294	52651	60337	69478

Table 5: Simplified example of power losses

For the simplified solution, the relative error accounts for 0.25% for the losses and 0.00072% for the total transmitted power. As the turbines are connected in a line, this is a worst case scenario and the results of the relative error should form an upper bound. Since the error is less than one percent and the simplification is much quicker to use, it is calculated in our algorithm with the simplified losses.

6.2. Remuneration of offshore wind energy

To obtain the financial loss the power loss causes, the remuneration of offshore wind energy in Germany [14] is used. In 2014 the new German Renewable Energy Sources Act (Erneuerbare-Energien-Gesetz – EEG 2014) came into force. One can choose between two opportunities:

- 12 years a remuneration of 15.4 ct/kWh and another 8 years 3.9 ct/kWh.
- 8 years a remuneration of 19 ct/kWh and another 12 years 3.9 ct/kWh

There are a few exceptions for wind parks very far away from the shore or in very deep water regions. Both is not the case for the Horns Rev wind farm.

In the calculations the first option is used. The average remuneration during the first twenty years per year is therefore:

$$\frac{15.4 \cdot 12 + 3.9 \cdot 8}{20} = 10.8 \quad [\text{ct/kWh}].$$

This refund is used to calculate the costs for the power losses.

Since the highest cost for the losses are caused by the ohmic losses, which proportional to the square of the current, one should actually use the root mean square of the output power of the turbines, to estimate the power loss. Information on this value is not available. Therefore the average power of an offshore turbine is used. In [6] it is estimated, that over the course of a year, an onshore wind turbine will typically generate about 24% of the theoretical maximum output and offshore generate 41%. The maximum power the turbines produce is 2 MW. Therefore the input power of

$$0.41 \cdot 2 = 0.82 \quad [\mathrm{MW}]$$

is used to calculate the power losses.

6.3. Esau-Williams' Algorithm Version 3

In every step the improvement of the costs for a different connection is checked and the costs of the losses are taken into account. Since both, cable costs and costs of the losses, can be given in Euro, it is easy to compare them. The data for the remuneration of offshore wind energy and thus the missing income the losses cause, can be found in section 6.2.

The interface for the input of the properties responsible for the power losses and the remuneration is shown in Figure 22.

```
%Information on power losses
AC_resistance=[0.16*10^(-3),0.079*10^(-3),0.041*10^(-3)];    % ohm per meter
tan_delta=0.0004;
capacity=[0.22*10^(-6),0.27*10^(-6),0.37*10^(-6)] ;           % farad per kilometer
frequency=50 ;                                                % 1 per second
period_of_validity=20;                                        % years
price_per_kWh=0.108;                                          % euro
root_mean_square_power_of_turbine=0.81*10^6;                  % watt
```

Figure 22: Input power losses

With this conditions implemented in the program with Esau-Williams' algorithm version 2, the layout shown on Figure 23 gets returned. For this example a runtime of approximately 15 seconds is obtained.

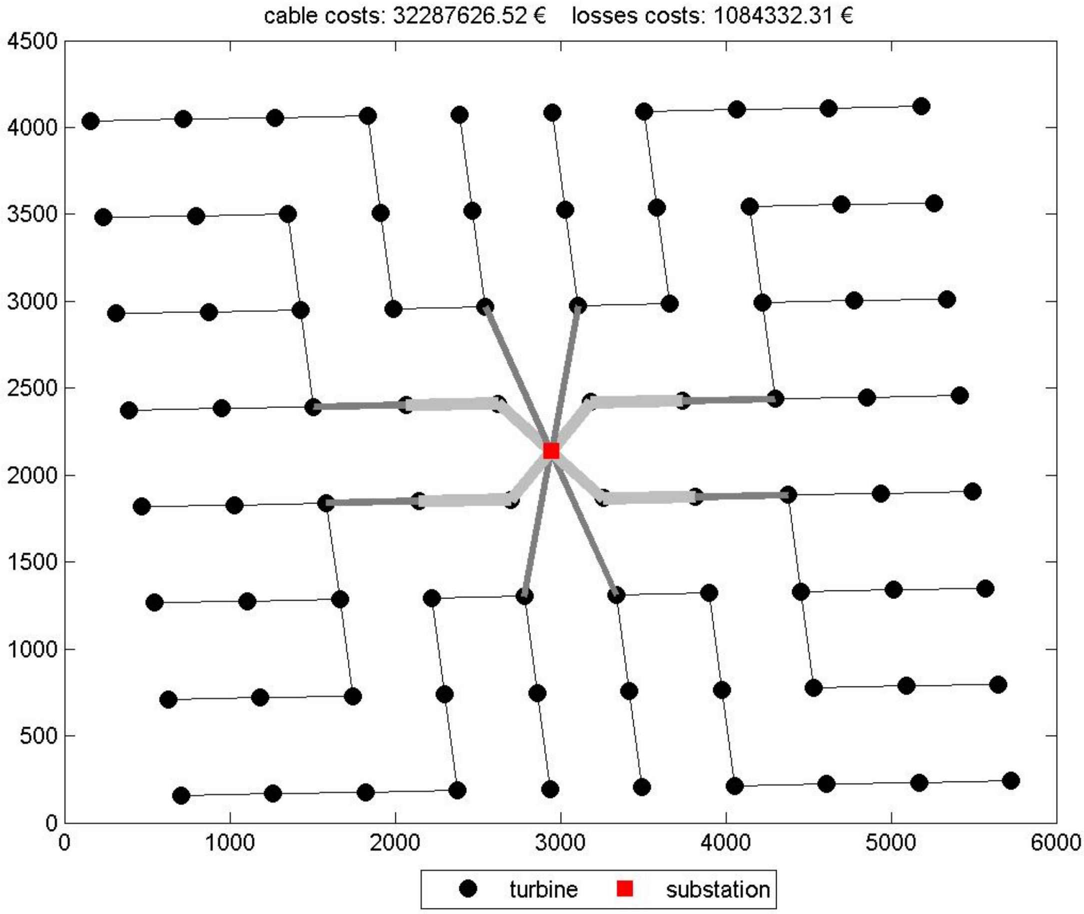

Figure 23: Input power losses

In order to determine whether or not the costs for the losses have actually been reduced, they are calculated for the example solution of version 2 in which the input data is the same as in this example. The algorithm actually optimizes the power losses! The results can be seen in Table 6.

Algorithm	cable costs	costs losses	overall costs
Esau-Williams' V2	$3.2365 \cdot 10^7$ €	$1.0883 \cdot 10^6$ €	$3.3454 \cdot 10^7$ €
Esau-Williams' V5	$3.2288 \cdot 10^7$ €	$1.0843 \cdot 10^6$€	$3.3372 \cdot 10^7$ €

Table 6: Comparison of Esau-Williams' Algorithm V2 & V3

Remarkable is that also the costs of the cables decrease in Version 3. A possible explanation for that is given in section 8.1.1. In section 8 more examples are given to determine if this example is only an exceptional case or if Version 3 optimises better in general.

7. Irregularities of the seabed

The irregularities of the seabed can be included in the models by refitting the distances between any two turbines with the actual distance considering the irregularities of the seabed.

A cable needs to be buried approximately one meter under the seabed surface. The seabed is not flat. Mountains and valleys under the water may therefore lead to a longer cable installation distance between turbines. If there is for example a hill between two turbines, it may be better not to choose the path straight over the hill, but a shorter path round the hill.

On Figure 24 the surface of the seabed where the Horns Rev Wind Park is placed is shown. The depth is given on a grid of 50 m. The maximal height distances is less than 10 meters.

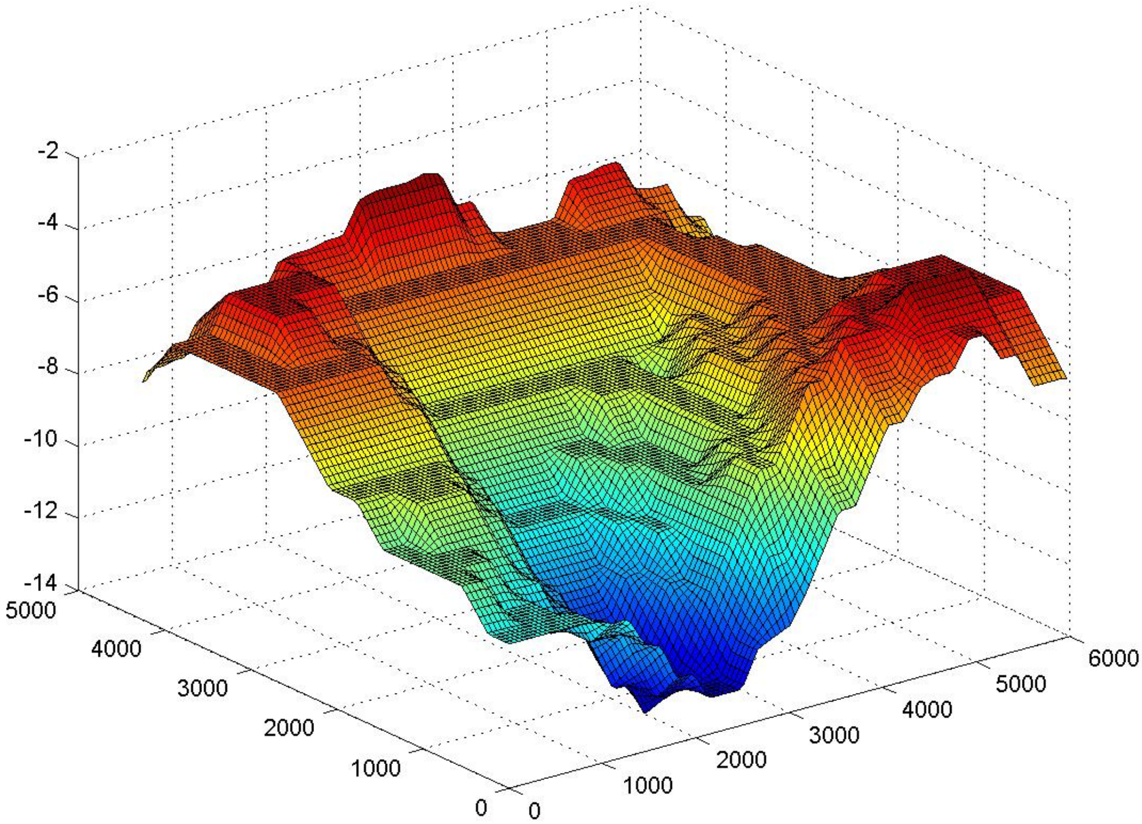

Figure 24: Horns Rev seabed surface

To consider the problem described above and to determine the best path between every two turbines, the seabed is regarded as a two dimensional grid graph with a lattice constant in x and y direction. At every grid point the depth of this point is given. To take the height differences into account, a connecting path is only allowed to directly connect neighbouring points. The blue points are the neighbouring points of the red point.

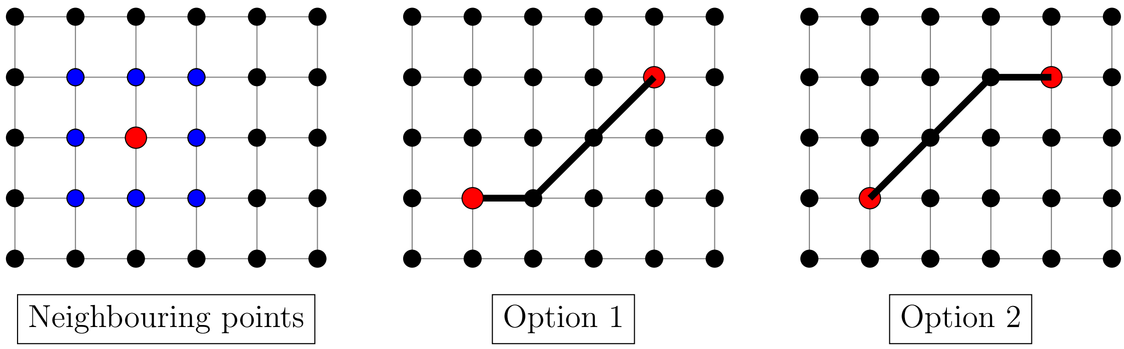

Figure 25: Path on a grid

In the center and on the left hand side of the Figure 25 two possibilities for a path connecting the red points are given. Without any height differences these two belong to the set of the shortest possibilities. The costs from one point to its neighbouring can be calculated with the Pythagorean theorem.

To apply an algorithm on this problem to find the shortest path between each pair of grid points we define a matrix "Distances" of the size n, which is the number of grid points. Distances(i, j) describes the distance between grid point i and grid point j. In the beginning every entry is initialized to infinity except for the entries to a direct neighbouring point. They are initialized with the calculated distance.

7.1. Floyd-Warshall Algorithm

To determine the length of the shortest path between every grid points, the Floyd-Warshall Algorithm is implemented [3]. The Algorithm

```
for all k ∈ {1..n}
    for all i ∈ {1..n}
        for all j ∈ {1..n}
            if Distance(i, k)+Distance(k, j) < Distance(i, j)
                Distance(i,j)=Distance(i, k)+Distance(k, j)
```

has a complexity of $\mathcal{O}(n^3)$.

Additionally the user is allowed to define a maximum installation depth, which is usually 40 m to 50 m below the sea level The depth of every grid point that is deeper than this upper bound is set to infinity. One must be careful that no turbine is located within an area where the depth is deeper than allowed. Additionally it must be ensured that every turbine can be connected by only using grid points whose depths are not deeper than allowed. Furthermore this method can be used to mark for example areas where no cable should go through, which can be used to leave some space for the shore connection.

Applying the Floyd-Warshall algorithm to the 50 m grid depth map of the area of the Horns Rev wind farm, takes a runtime of approximately 20 hours. The smaller the lattice constant, that defines the distance to the next grid point in x and y direction, the longer the runtime. In the used example there are 10 320 grid points. However for every area this needs to be done only once and for different placings of the turbines the already computed data can be used, so the runtime is acceptable.

More important is the performance identifying every turbine with a grid point and transferring the data for the distances of those points. By using MATLAB and the example of the Horns Rev wind farm this takes 13.7 seconds. Most of the time (13.4 seconds) is spent on loading the matrix in which the distances are saved. The saving and loading of this matrix is done with the MATLAB methods save() and load().

The results applying Esau-Williams' algorithms V2 and V3 on the 3D data are shown in Figure 26 and Figure 27. The depth of the sea is indicated by the colour in the background. The kind of Floyd Warshall algorithm that is implemented does not return the path from one turbine to another, only the distance. The output our programme needs to supply are the optimised costs for cables and losses and not how exactly the layout looks like. Therefore the exact cabling between any two turbines is not shown.

The runtime including the time to identify the turbines and get their 3D distances is for the Esau-Williams' algorithm V2 approximately 21 seconds and for the Esau-Williams' algorithm V5 approximately 28 seconds.

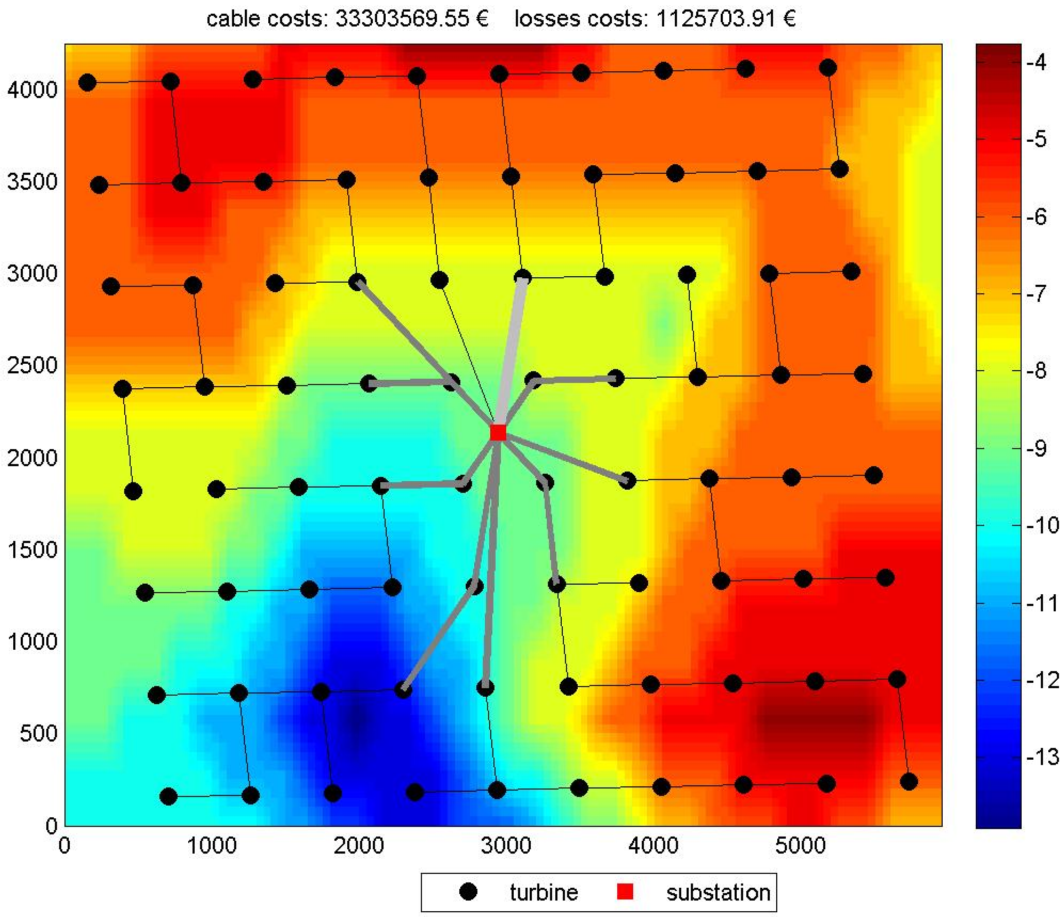

Figure 26: Esau-Williams' Algorithm V2 applied to Horns Rev wind park using 3D data

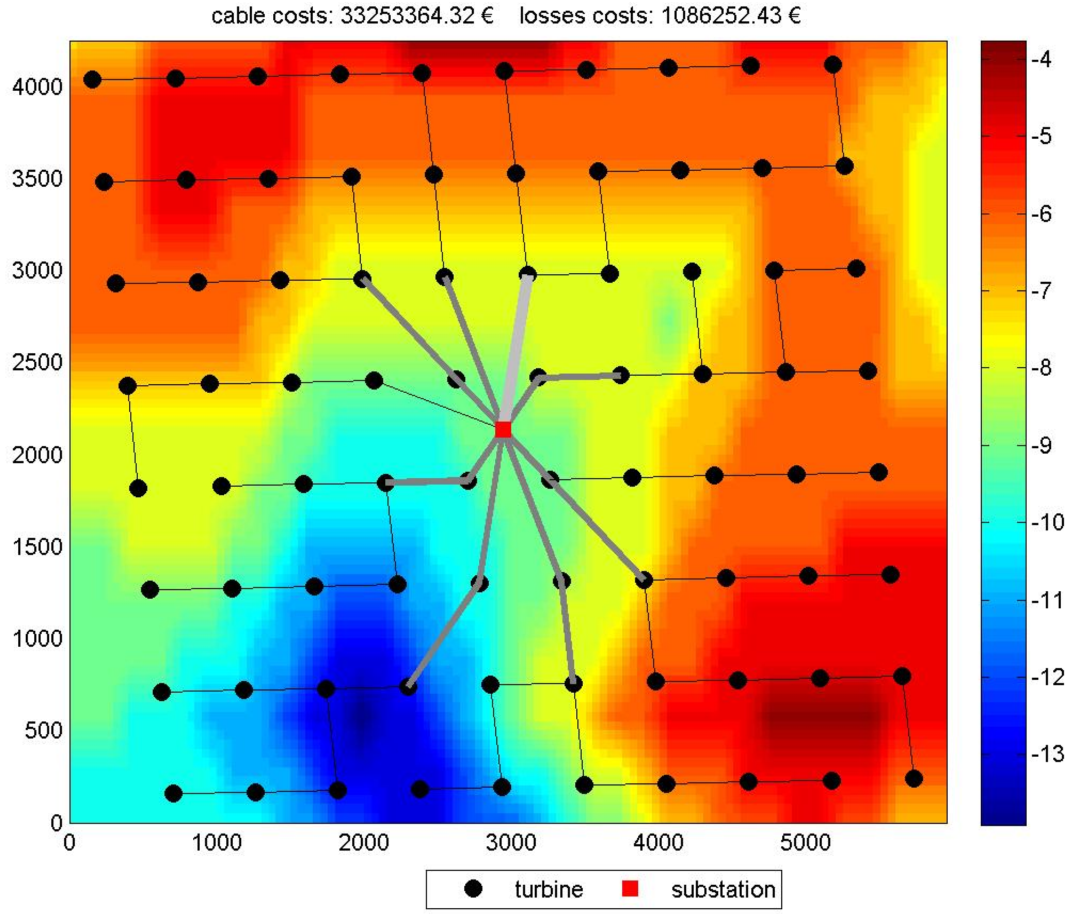

Figure 27: Esau-Williams' Algorithm V3 applied to Horns Rev wind park using 3D data

8. Accuracy of the results

In this section an error estimation is performed. Furthermore the two algorithms are compared by calculating more examples. Additionally the results provided by the implemented algorithms are compared to the actual layout.

8.1. Accuracy of the solutions

The estimation of the error can be divided into two parts: the accuracy of the Esau-Williams' algorithm itself and the consideration of the irregularities of the seabed.

8.1.1. Accuracy of the Esau-Williams' Algorithm

In [10] the optimizing accuracy of 15 different positions of 80 nodes and the substation are determined. The average optimisation accuracy for the Esau-Williams' algorithm is about 5 %. However in this work only one type of cable is allowed. It can be assumed that in the case of more than one cable type the average error increases since there are more possibilities for wrong decisions.

Next there is an example given that makes it possible to construct a layout in which the costs for the computed solution of Esau-Williams' Algorithm will be 1.5 times as high as for the optimal one.

Suppose a cable that is able to connect three turbines is available. Esau-Williams' algorithm will compute layout (a) of Figure 28 whereas the optimal solution is Layout (b).

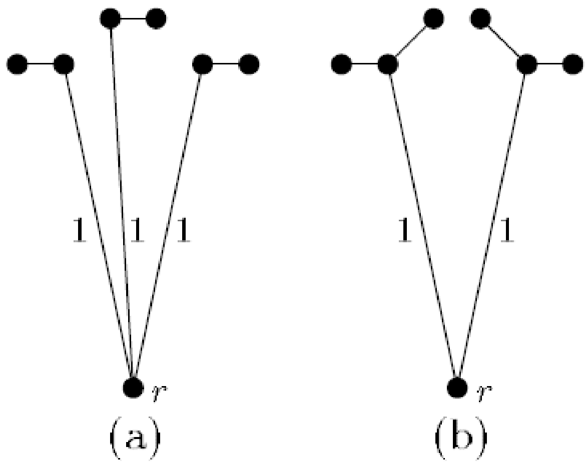

Figure 28: Example Esau-Williams' algorithm's result vs. optimal solution

Thus it is not possible to state the relative error of the algorithm. But since in this case turbines are very unevenly distributed, it is unlikely that such a situation will come up in a wind farm.
Version 3 of the implemented algorithm including the losses, ameliorates sometimes the costs of the cables. This algorithm can be compared to the modified Esau-Williams' algorithm described in [10]. The savings are multiplied by *weights* in a way that savings of sub trees with a greater number of included nodes are less worth. The costs for losses can be compared with the weights since they decrease with the number of nodes in a sub tree. This modification improves the result in most of the cases. However the differences between them are too great to transfer those numbers to our algorithm.

8.1.2. Accuracy of the treatment of the irregularities in the seabed

To take the irregularities of the seabed into account turbines are identified with grid points. Every grip point is associated with an square with the area of 50x50 m^2 in which the turbines are identified with this grid point. Thus the maximal error caused by identifying a turbine is half the length of a diagonal of this square: $1/2 \cdot \sqrt{2} \cdot 50$ m. By assuming the actual distance between two turbines to be 500 meters, the worst case error would be $\sqrt{2} \cdot 50$ m which is approximately 17% of 500 m. This can be reduced by using a finer grid to model the seabed.

Furthermore the path from one grid point to another does not represent the distance correctly. The worst-case for the error is shown in Figure 29.

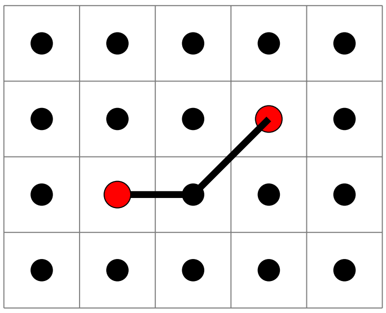

Figure 29: Worse-case path on a grid

By assuming the seabed to be flat and the lattice constant to be 50 meters, the actual length between those points is

$$\sqrt{100^2 + 50^2} \approx 111.8 \text{ [meter]},$$

whereas the algorithm computes

$$\sqrt{2} * 50 + 50 \approx 120.71 \text{ [meter]}.$$

The relative error is 7.8%.
This error can be increased by height differences. However the seabed of the area in

which a wind farm is planned is normally not too uneven, compare e.g. Horns Rev wind farm. Therefore the magnitude of this error will not change too much.

The overall worst case error on the lengts for the part in which the irregularities of the seabed are computed is therefore approximately 15 %.

8.2. Esau-Williams' Algorithm V2 vs. V3

To get an idea how much the Esau-Williams' Algorithm V3 reduces the costs for the power losses compared to algorithm V2, they are applied on some further examples. The turbine positions of the Horns Rev wind farm provide still the basis for all the examples. Examples in which the turbine is located in the center are named C. Otherwise they are named A, for the actual turbines system as it is shown in Figure 30. If the irregularities are considered in the example it is marked with an i behind the C or A. Finally the used cable types are listed behind these abbreviations. They are numbered as in Table 3 in section 6.1.1. An example in which the substation is in the center, the irregularities are taken into account and cable type 1,5,6 is used is called Ci156. The costs in Euro as well as the saving percentages are listed in Table 7.

	Example	Cable costs	%	Costs losses	%	Overall costs	%
V2	C147	$3.2365 \cdot 10^7$ €		$1.0883 \cdot 10^6$€		$3.3454 \cdot 10^7$ €	
V3	C147	$3.2288 \cdot 10^7$ €	0.24	$1.0843 \cdot 10^6$€	0.37	$3.3372 \cdot 10^7$ €	0.25
V2	C125	$3.2627 \cdot 10^7$ €		$1.2746 \cdot 10^6$€		$3.3901 \cdot 10^7$ €	
V3	C125	$3.1862 \cdot 10^7$ €	2.34	$1.0945 \cdot 10^6$€	14.13	$3.2956 \cdot 10^7$ €	2.79
V2	C37	$3.3572 \cdot 10^7$ €		$8.8207 \cdot 10^5$€		$3.4454 \cdot 10^7$ €	
V3	C37	$3.4721 \cdot 10^7$ €	-3.42	$7.2454 \cdot 10^5$€	17.86	$3.5445 \cdot 10^7$ €	-2.88
V2	Ci234	$3.4700 \cdot 10^7$ €		$1.2512 \cdot 10^6$€		$3.5951 \cdot 10^7$ €	
V3	Ci234	$3.4606 \cdot 10^7$ €	0.27	$1.1590 \cdot 10^6$€	7.37	$3.5765 \cdot 10^7$ €	0.52
V2	Ci7	$4.3949 \cdot 10^7$ €		$5.9498 \cdot 10^5$€		$4.4544 \cdot 10^7$ €	
V3	Ci7	$4.4049 \cdot 10^7$ €	-0.23	$6.6429 \cdot 10^5$€	-11.65	$4.4714 \cdot 10^7$ €	-0.38
V2	A147	$3.5341 \cdot 10^7$ €		$1.1051 \cdot 10^6$€		$3.6446 \cdot 10^7$ €	
V3	A147	$3.5539 \cdot 10^7$ €	-0.56	$1.0501 \cdot 10^6$€	4.98	$3.6589 \cdot 10^7$ €	-0.39
V2	A26	$3.7236 \cdot 10^7$ €		$1.1091 \cdot 10^6$€		$3.8345 \cdot 10^7$ €	
V3	A26	$3.7729 \cdot 10^7$ €	-1.32	$9.9663 \cdot 10^5$€	10.14	$3.8726 \cdot 10^7$ €	-0.99
V2	A125	$3.5614 \cdot 10^7$ €		$1.2390 \cdot 10^6$€		$3.6853 \cdot 10^7$ €	
V3	A125	$3.5197 \cdot 10^7$ €	1.17	$1.0699 \cdot 10^6$€	13.65	$3.6267 \cdot 10^7$ €	1.59
V2	A257	$3.6728 \cdot 10^7$ €		$1.1420 \cdot 10^6$€		$3.7871 \cdot 10^7$ €	
V3	A257	$3.6725 \cdot 10^7$ €	0.01	$1.0019 \cdot 10^6$€	12.27	$3.7727 \cdot 10^7$ €	0.38
V2	Ai6	$4.7602 \cdot 10^7$ €		$1.3030 \cdot 10^6$€		$4.8905 \cdot 10^7$ €	
V3	Ai6	$4.6461 \cdot 10^7$ €	2.40	$1.2949 \cdot 10^6$€	0.62	$4.7756 \cdot 10^7$ €	2.35
V2	Ai27	$4.3087 \cdot 10^7$ €		$1.5311 \cdot 10^6$€		$4.4618 \cdot 10^7$ €	
V3	Ai27	$4.2551 \cdot 10^7$ €	1.24	$1.6714 \cdot 10^6$€	-9.16	$4.4222 \cdot 10^7$ €	0.89

Table 7: Further examples of Esau-Williams' Algorithm V2 & V3

The average of the savings by creating the layout with Version 3 instead of Version 2 is 0.19 % for the cable costs, 5.51 % for the costs of the losses and 0,38 % for the overall costs . Since the cable costs are approximately 30 times higher than the costs for the losses the improvement of the losses is barely noticeable in the overall costs.

8.3. Comparison to the actual Horns Rev layout

The aim of this subsection is to compare the layout results of the implemented algorithms to the actual cable layout of the Horns Rev 1 wind farm.

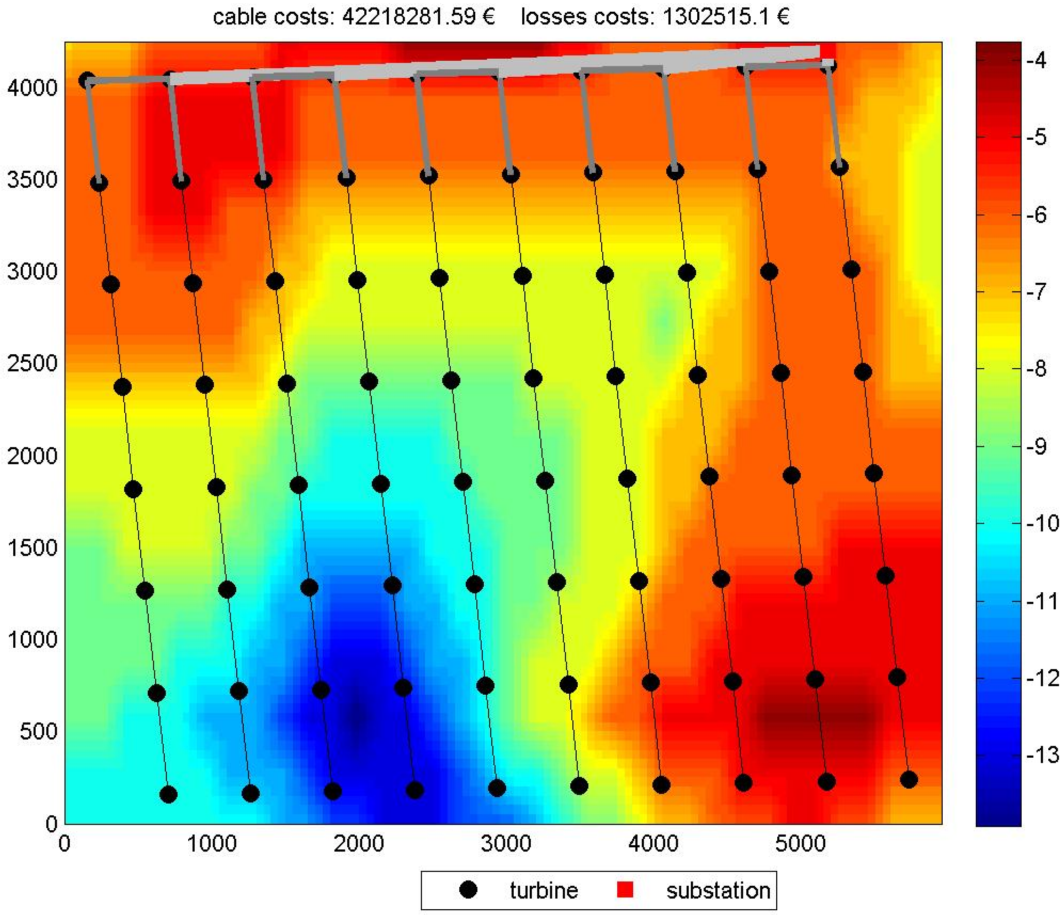

Figure 30: Actual Horns Rev

The actual layout is shown in Figure 5. A cable which is able to conduct the power of 16 turbines is used. Since no information on such a cable is known the needed values are approximated:

- With the formulas and information of section 5.2 the current rating of cable connecting 16 turbines is calculated to be at least

$$55.56 \cdot 16 \approx 888.96 \text{ [A]}.$$

  As a simplification the value of a cable with a cross section area of 3 x 630 mm^2 is used as a comparison. The cost of one capacity of current is

$$\frac{\text{Price per meter}}{\text{Maximal current}} = \frac{400}{721} \approx 0.55 \text{ [€/A·m]}$$

  The price per meter for cable connecting 16 turbines can therefore be approximated by

$$888.96 \cdot 0.55 \approx 489 \text{ [€/m]}.$$

- By fitting the values of the seven different cable sizes for the AC resistance (f) on the Maximal current (x), the fit method of MATLAB returns

$$f(x) = 0.0008017 \cdot e^{-0.004229 \cdot x}.$$

  As the mean squares of errors on f(x) is smaller than $4.5 \cdot 10^{-12}$, the fit seems to be realistic. The function returns an AC resistance of $2 \cdot 10^{-5}$ Ω per meter for a cable with a current rating of 888.96 A.

- The capacity value used for the calculation of the dielectric loss is for the thinnest cable 0.22 μF/km. With every turbine this value increases by approximately 0.025 μF/km. Therefore the cable connecting 16 turbines has a capacity of 0.45 μF/km.

Cables that are able to connect 16, 8 and 6 turbines are used to readjust the actual layout of the Horns Rev as it is shown on Figure 30. The location for the turbines corresponds to the actual one. The substation is placed approximately at the location of the actual substation.

The algorithms Esau-Williams' V2 and V3 are also applied using the same cables and location for the substation. The layouts are shown in Figure 31 and Figure 32.

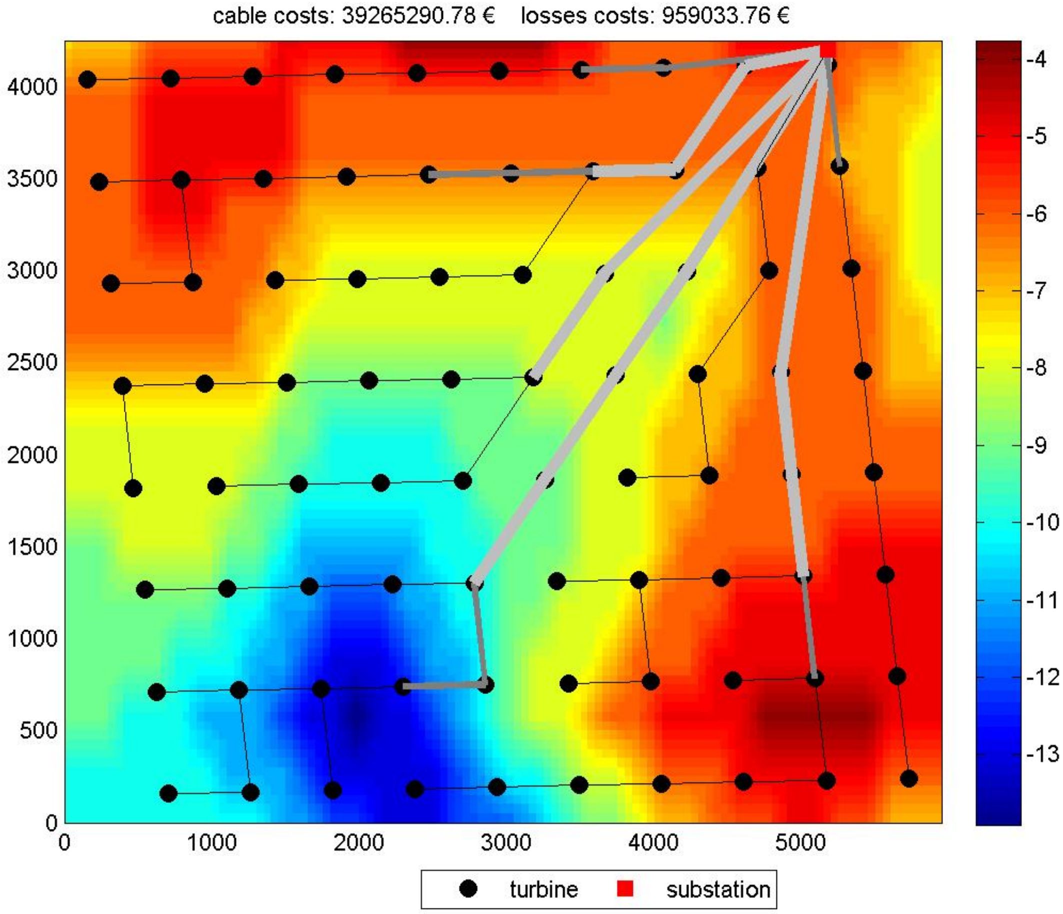

Figure 31: Actual cabeling of Horns Rev wind farm

Algorithm	cable costs	costs loss	overall costs
Actual cable layout	$4.2218 \cdot 10^7$ €	$1.3025 \cdot 10^6$ €	$4.3521 \cdot 10^7$ €
Esau-Williams' V2	$3.9265 \cdot 10^7$ €	$9.5903 \cdot 10^5$ €	$4.0224 \cdot 10^7$ €
Esau-Williams' V3	$3.9208 \cdot 10^7$ €	$9.8434 \cdot 10^5$ €	$4.0192 \cdot 10^7$ €

Table 8: Comparison of Esau-Williams' algorithm V2 & V3 to the actual cabling layout

Sometimes cables are superposed. In reality these cables need to be moved a bit so that they can be installed parallel next to each other. This does not falsify the results, since a shift of a few meters is enough.

The costs for cables, losses and the overall costs, cf. Table 8, are lower for the layout solutions of the algorithms than the already built layout. The savings for algorithm *Version 2* are 7.68 % and for algorithm *Version 3* 7.65 %!

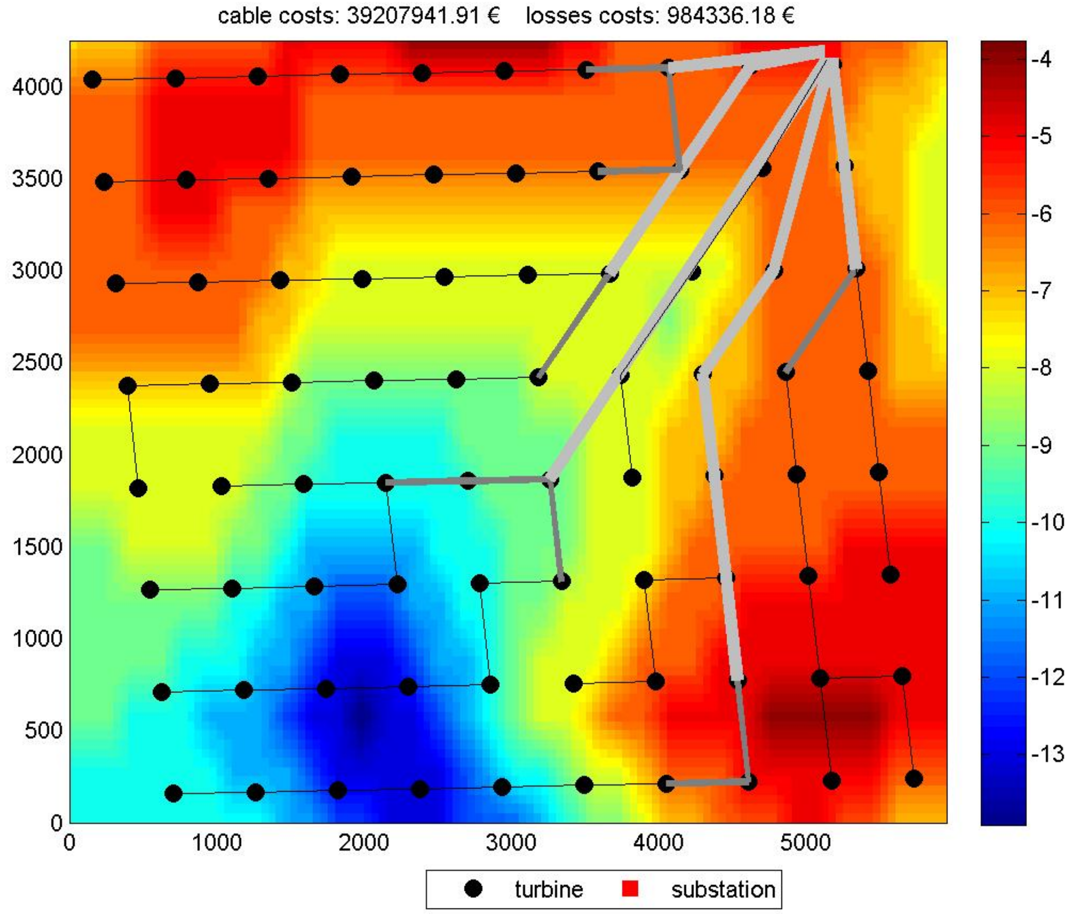

Figure 32: Esau-Williams' algorithm V3 applied on the actual substation position of the Horns Rev wind farm

9. Outlook & Conclusion

Although the constructed algorithms do already consider a lot of features for a cabling layout of a wind farm, some more ideas came up that might make the layout even more optimized and realistic. However elaborating these concepts would go beyond the scope of this bachelors thesis and is therefore left for future work. The ideas are listed below, some with approaches towards the solution.

- Since the costs for the cables is in average 30 times higher than the costs for the losses, it is recommendable for future work to concentrate on the costs for the cables. It would be interesting to modify some more algorithms that give heuristics for the capacitated minimum spanning tree. Some possible matching candidates are already found:
  - The first possibility is to improve the layout of the already implemented algorithm returns. One could do this by calculating the regret costs as defined in section 3.1 with the difference that all possible connections are considered and not only those directly connected to the substation. Another possibility is to use one of the algorithms described in [7]. In [7] two algorithms of the same concept are described. Both are using a “node-based, multiexchange neighborhood structure” which was originally proposed by Ahuja et al. (2001). To use this algorithm first one needs to construct a start tree, In which randomly computed minimum spanning trees can be used. In the next steps one constructs series of nodes belonging to different sub trees and check if it is worth to change them. A possible changing series is shown in fig. 33. This is repeated until no changing would lead to cheaper costs.

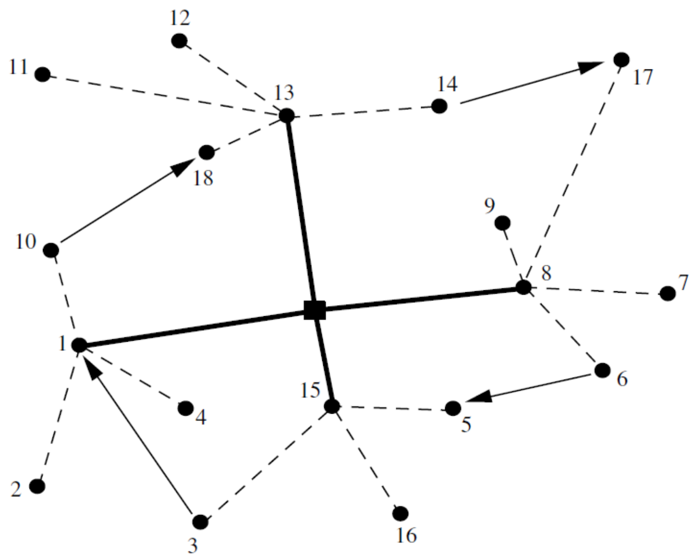

Figure 33: Example: One step of the local search algorithm, cf.[7]

For 100 nodes this kind of algorithms have an average error of 5.32 %, the first one has a running time of about 200 seconds and the other one of runs over 3000 seconds. Therefore this algorithm is only suitable to be used on an already optimized layout. To build up a layout using this algorithm would use too much time.

- The in [7] described construction heuristic also has a short runtime and can therefore be used to solve this problem:
  * first step is to directly connect every node to the substation
  * take the cable with the highest capacity. For every node i calculate the costs for changing the cable to the substation with the higher capacity cable connecting a number of nodes which do not exceed the capacity of the cable to node i. Try every possibility for the nodes to connect to the node i and choose the cheapest one. If the costs all together are cheaper than the original costs, change the spanning tree to this solution
  * repeat the previous step by only taking the nodes into account that are directly connected to the substation until the saving costs are negative.
  * repeat the two steps above by using the next lower capacity, regarding the sub trees that are already connected with a thicker cable as separate trees with the connection node as substation
  * stop if even with the second lowest capacity the cost can not be improved any more

  An example of the construction heuristic algorithm can be seen on Figure 34. The first step is shown on the left picture. The right picture shows the layout after the improvement with the cable of capacity 10.

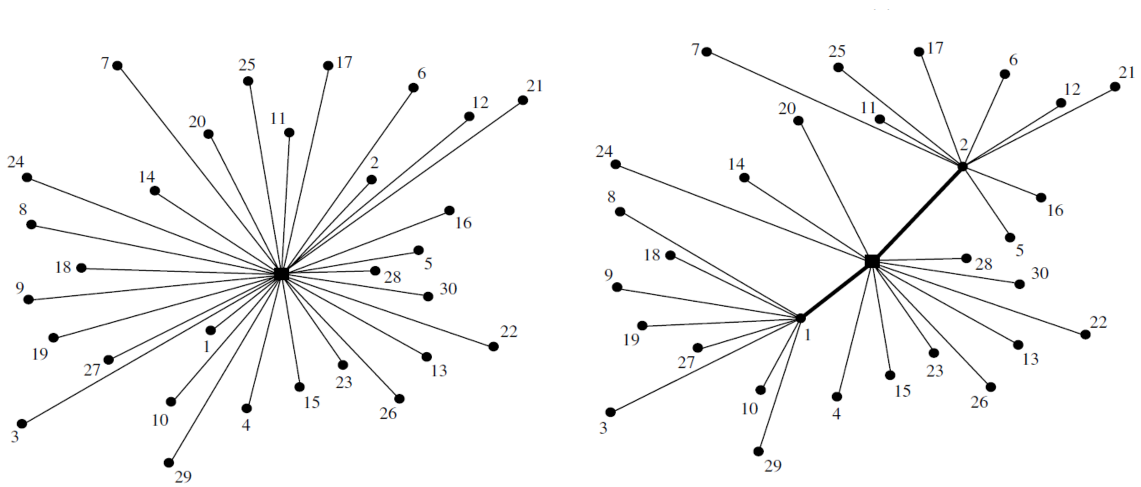

Figure 34: Example of the constuction heuristic algorithm, cf. [7]

In [7] the algorithms are tested for different possible numbers of nodes and arrangements. The solution of this algorithm for 100 nodes has an average error of 8.2% and a runtime of 0.04 seconds.

  - Furthermore the in section 8.1.1 mentioned modified Esau-Williams' algorithm that includes weights in its calculation seems to be a suitable candidate. It is worked out in [10].

- Most of the turbines have two cables connected to them. There are further costs for electrical switch gears and cable protecting systems if more than two cables are connected to a turbine and less if only one cable is connected to a turbine. These costs do not balance themselves. Furthermore every last turbine needs an external source of electricity, to be able to execute basic required functions in cases of damage: They always need to be able to turn the turbine out of the wind and the security light for air planes needs to shine in the night. This source can be either an external generator or a connection to another turbine that can if needed deliver supply current [24]. To give a more realistic presentation on the problem these costs should be considered. It would also be reasonable to update the algorithm with these aspects. The extra costs could be subtracted of the regret costs by connecting a turbine to another that already has two turbines connected to itself. This solution can be compared to a solution that does not allow branching of the cables. In [18] two algorithms calculating a non branching layout are compared. These are shortly described:

  - Different kinds of greedy algorithms:
    - start at substation, connect nearest turbine from this turbine to nearest turbine until length of string is reached
    - start again from substation
    - algorithm tries to keep down the number of strings
    - (in this thesis they use only one cable type, which sometimes does not lead to the shortest cable connection)
    - also some variations of this methods are given
  - Genetic algorithm

- The seabed is actually not static but dynamic. Due to for example sand wave effects the needed cable burial depth can vary. To install the cable deeper in the ground higher costs are guaranteed. The master thesis [18] mainly deals with these effects. Also different soil textures can cause higher installation costs. One idea to include them is to update the algorithm that considers the irregularities of the seabed and returns the lengths to connect the turbines. The higher prices for different soils could be recalculated in lengths and the actual algorithms does not have to be changed.

The goal setting of finding an algorithm that improves the costs for cables and the power losses is reached. However it is found out that it makes more sense to concentrate only on the cable costs. The share of the power losses is to small to effect the overall costs essentially.

Even though a few features are still missing in the simulation and optimization for the cabling layout, the main, expensive aspects are considered.

The algorithm provides a more than 7 % cheaper cabling layout for the original Horns Rev wind farm when turbines and substation are placed at the same! This is a great result regarding the short runtime. With the in this thesis determined algorithm only some seconds would have been required to save more than 3,000,000.00 €. Some more research on this subject is worthwhile.

A. Appendix: Data of Horns Rev wind farm

x	y
703.881	157.563
626.070142857	711.759714286
548.259285714	1265.95642857
470.448428571	1820.15314286
392.637571429	2374.34985714
314.826714286	2928.54657143
237.015857143	3482.74328571
159.205	4036.94
1262.01311111	166.829333333
1184.20225397	721.026047619
1106.39139683	1275.2227619
1028.58053968	1829.41947619
950.76968254	2383.61619048
872.958825397	2937.81290476
795.147968254	3492.00961905
717.337111111	4046.20633333
1820.14522222	176.095666667
1742.33436508	730.292380952
1664.52350794	1284.48909524
1586.71265079	1838.68580952
1508.90179365	2392.88252381
1431.09093651	2947.0792381
1353.28007937	3501.27595238
1275.46922222	4055.47266667
2378.27733333	185.362
2300.46647619	739.558714286
2222.65561905	1293.75542857
2144.8447619	1847.95214286
2067.03390476	2402.14885714
1989.22304762	2956.34557143
1911.41219048	3510.54228571
1833.60133333	4064.739
2936.40944444	194.628333333
2858.5985873	748.825047619
2780.78773016	1303.0217619
2702.97687302	1857.21847619
2625.16601587	2411.41519048
2547.35515873	2965.61190476
2469.54430159	3519.80861905
2391.73344444	4074.00533333

x	y
3494.54155556	203.894666667
3416.73069841	758.091380952
3338.91984127	1312.28809524
3261.10898413	1866.48480952
3183.29812698	2420.68152381
3105.48726984	2974.8782381
3027.6764127	3529.07495238
2949.86555556	4083.27166667
4052.67366667	213.161
3974.86280952	767.357714286
3897.05195238	1321.55442857
3819.24109524	1875.75114286
3741.4302381	2429.94785714
3663.61938095	2984.14457143
3585.80852381	3538.34128571
3507.99766667	4092.538
4610.80577778	222.427333333
4532.99492063	776.624047619
4455.18406349	1330.8207619
4377.37320635	1885.01747619
4299.56234921	2439.21419048
4221.75149206	2993.41090476
4143.94063492	3547.60761905
4066.12977778	4101.80433333
5168.93788889	231.693666667
5091.12703175	785.890380952
5013.3161746	1340.08709524
4935.50531746	1894.28380952
4857.69446032	2448.48052381
4779.88360317	3002.6772381
4702.07274603	3556.87395238
4624.26188889	4111.07066667
5727.07	240.96
5649.25914286	795.156714286
5571.44828571	1349.35342857
5493.63742857	1903.55014286
5415.82657143	2457.74685714
5338.01571429	3011.94357143
5260.20485714	3566.14028571
5182.394	4120.337

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