Diese Arbeit wurde vorgelegt am
Lehrstuhl für Mathematik (MathCCES)

Optimale Verkabelung von Heliostaten in Solarturmkraftwerken

Shortest Cable Routing of Heliostats in Solar Tower Power Plants

Bachelorarbeit
Mathematik

September 2017

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Aachen, im September 2017

Franziska Freya Ossenbrink

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1 Introduction

1.1 Motivation

Fighting climate change is one of the most important and also difficult challenges our society has to deal with nowadays. The awareness of global warming in people's minds is luckily rising and there already have been made concrete plans to reduce greenhouse gas emissions on a national and international level. In the end of 2016, Germany adopted its *Climate Action Plan 2050* which aims to reduce the greenhouse gas emissions in the next thirty years significantly. It is a measure to fulfill the commitments Germany has made under the *Paris Climate Agreement* that has been signed by 195 countries in June 2017. Its aim is to keep the global average temperature from rising more than 2 Kelvin.

To reduce greenhouse gas emissions we need to stop using fossil fuels to generate power and proceed to use renewable energies instead. Next to wind power produced e.g. in on- or offshore wind farms is solar radiation an inexhaustible source to generate power. Photovoltaic systems are already widely spread and can be seen as solar parks or private on rooftops in many countries.

Another way to use solar power is through concentrated solar power systems that use mirrors to concentrate solar radiation onto a small area and thus generate thermal power such as parabolic trough plants or solar tower power plants. These power plants have the advantage of having the possibility to save the produced energy thermally for several days and thus are a supplement to other renewables.

This thesis deals with solar tower power plants. The aim is to optimize the cabling used in the field around the solar tower to reduce the costs for prospective power plants of this type. As a consequence renewables could be provided faster and thus contribute in fulfilling the climate action plans that have been made.

1.2 Definition of the Problem

Solar tower power plants consist of up to several thousands of flat mirrors, so called heliostats, reflecting the solar radiation to the top of a tower where a receiver is located, as shown in Figure 1. As a result of the high concentration of solar radiation, the receiver heats to temperatures of several hundred degrees. The heat transfer medium perfusing the receiver, mostly water, molten salt or air, then exchanges the heat to steam which finally powers a turbine located at the bottom of the tower generating electricity.

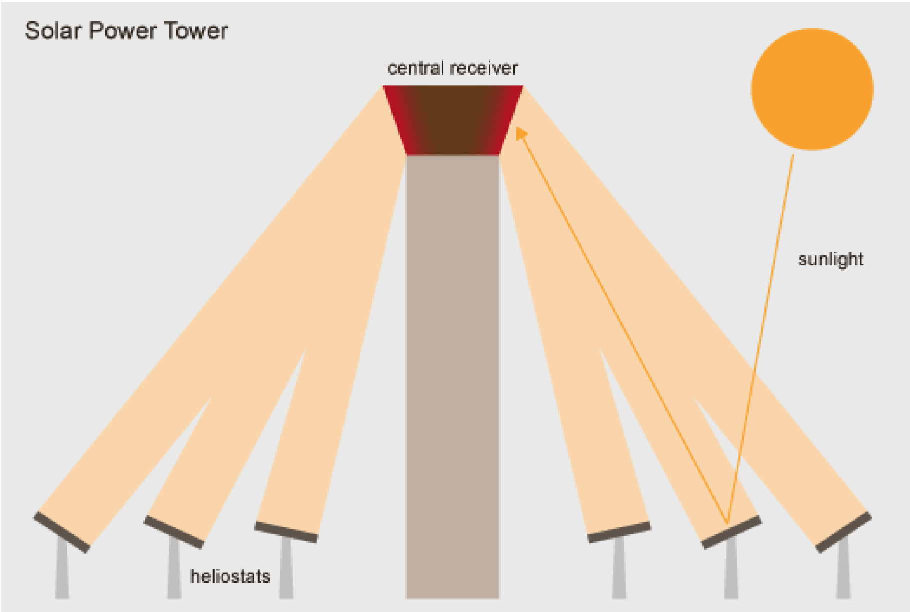


Figure 1: Layout of solar tower power plants, cf. [22]. The solar radiation is reflected by heliostats to a central receiver that is located at the top of the solar tower.

As one can see in Figure 2, the construction of the heliostat field makes up to one third of the total costs of the power plant. Even though the receiver is placed in the upper section of the tower, the placement of the mirrors may lead to individual mirrors being blocked or shaded, affecting the efficiency and costs of the power plant. Therefore, to maximize the energy output the heliostats track the sun and adjust their orientation every ten seconds. To be able to rotate, each mirror has to be provided with power and receive the data to which position it has to change next.

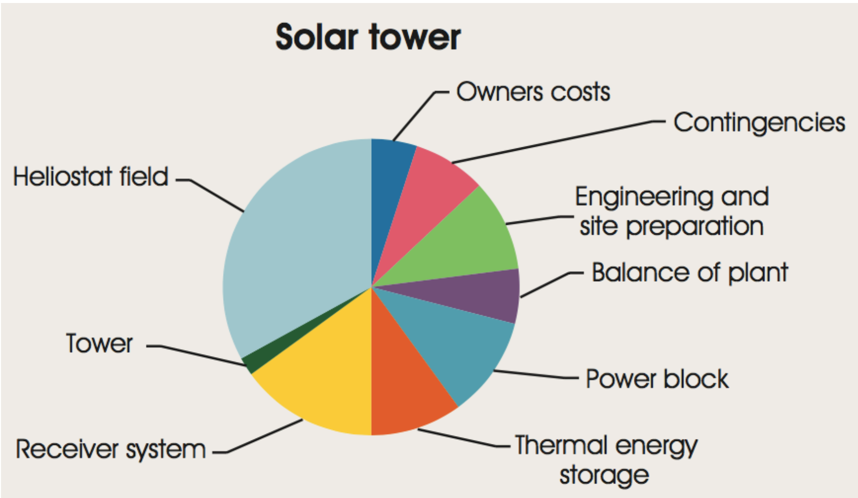


Figure 2: Total costs of a solar tower power plant in South Africa, cf. [17]. The heliostat field including cabling aggregates approximately one third of the total costs.

A possible option is to consider each mirror as a self-sufficient system meaning that it provides its own power by having a photovoltaic system installed. In this case the data is imparted by wireless LAN. This option is rather expensive since solar tower power plants consist of several hundreds of heliostats as stated above so that equipping each mirror with photovoltaics adds up to a high amount of money. Besides, iron used for the construction of the heliostats would shield the LAN signal so that it cannot be guaranteed that it reaches each of them.

In this thesis we want to focus on another option. Each heliostat is connected to the tower by a power cable for the motor to track the sun and a data cable for control. Our aim is to find the optimal cable layout within the heliostat field meaning to reduce the costs by using as few cable meters as possible while considering several constraints which will be presented in further detail in Section 2 and Section 5.

Our aim is to optimize the cabling layout of a huge heliostat field being composed of 12 676 heliostats that are arranged in various circles around the tower, see Figure 3. Since the information about the coordinates of the huge field were provided by German Aerospace Center (DLR) we will refer to it as DLR Field. To determine the most suitable method both for the data and the power cable we will test a set of algorithms on a smaller field named PS10 and compare our results with the costs of the actual layout. The best algorithms will be chosen and applied to the huge field. Since today's solar power plants tend to be equipped with several thousands of mirrors we try to make our model as efficient as possible and pay attention to the running time.

Europe's first commercial solar tower power plant, named PS10 Solar Power Plant, is placed near Sevilla in Spain and operates since the beginning of 2007. Compared to current power plants it is rather small containing 624 heliostats and producing power of at most 11 megawatt. It has an appropriate size to test our algorithms and to finally choose the best one for the data and the power cable.

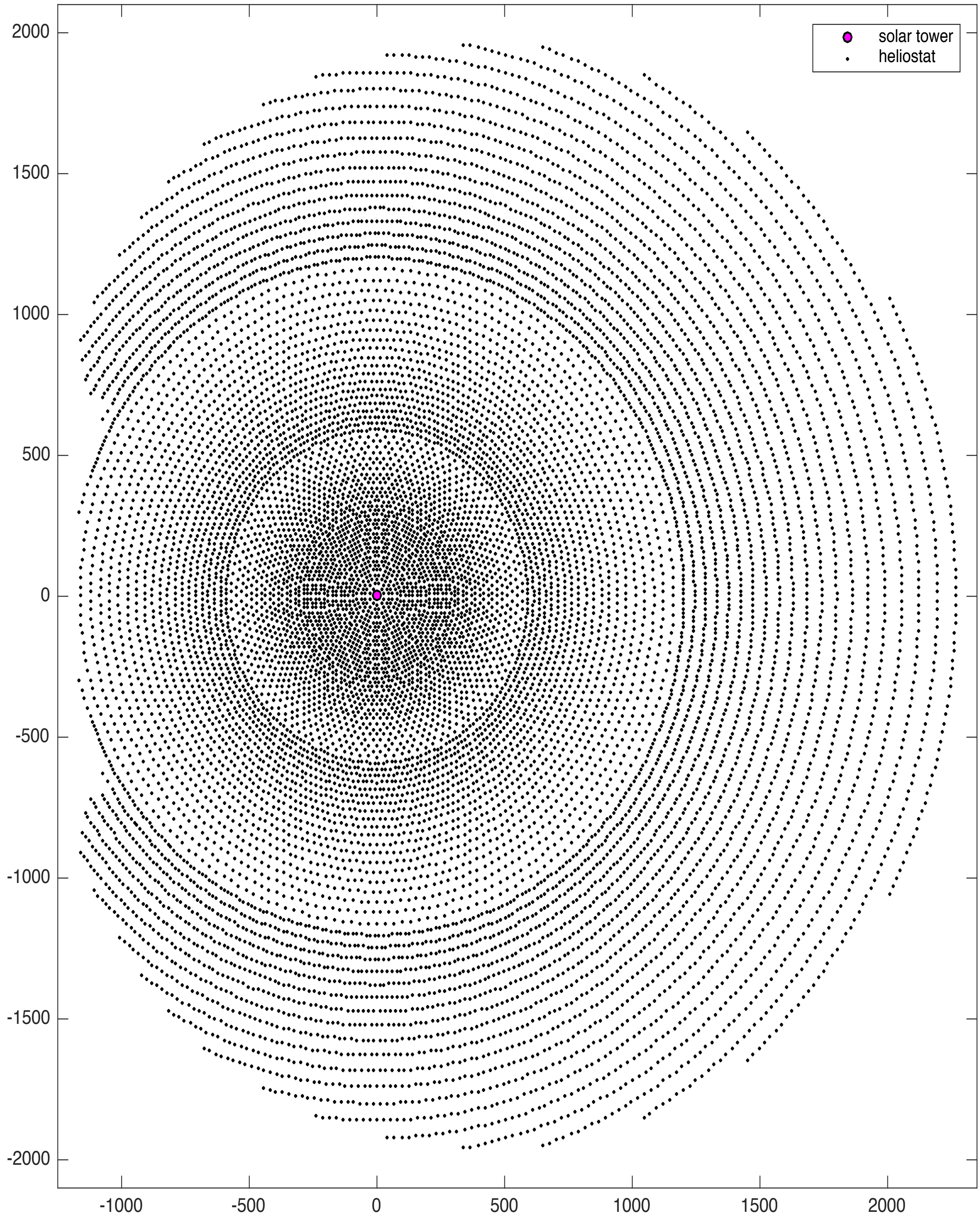


Figure 3: Solar tower: DLR Field. This huge solar tower power plant consists of 12 676 heliostats and a centrally placed tower.

1.3 Preliminary Work

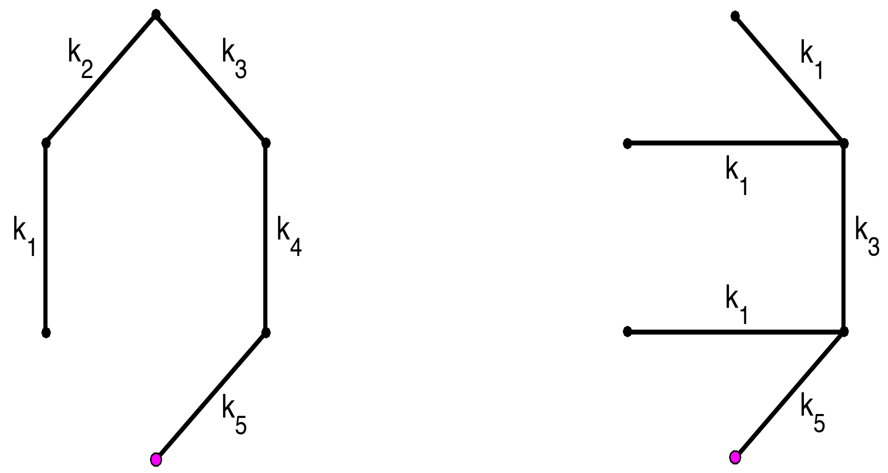
While researchers already deal with the problem of finding the optimal layout of heliostat fields, cf. [15], the optimization of cabling in these fields is rather new. However, there are researchers working on finding the optimal power cable layout in offshore wind farms, cf. [1, 18]. Even if considerably less nodes have to be connected – offshore wind farms mostly consist of no more than 150 wind turbines while today's solar tower power plants consist of up to thousands of heliostats – the progress is worth mentioning at this point since approaches of graph theory have been made to which we want to refer in this thesis.

In offshore wind farms power is directly produced by the turbines and has to be carted away by power cables. Thus different types of cables are used dependent on the power that is transmitted by the turbines being connected to them. Hence each cable type has a maximum capacity of wind turbines that it is able to connect. In contrast heliostats in solar tower power plants have to be provided with power since they are equipped with a motor to track the sun. We will consider different capacities in our model as well. For example there are different types of power cables each having another capacity dependent on its cross section as will be explained in detail in Section 5.

Most approaches try to find an optimal routing layout meaning that each wind turbine has one input and at most one output cable such as exemplary shown in Figure 4a. In [1] the problem has been described as an Open Vehicle Routing Problem with unit demands and additional planarity constraints, meaning that a given set of nodes shall be visited during a tour and being connected to a depot. Each route shall not exceed the vehicle capacity and is not allowed to cross with the other routes. The aim is to minimize the total costs that occur for this exact route.

Furthermore, some approaches allow branching of cables and thus compose a layout having a tree structure (see Figure 4b). Comparing both layouts of Figure 4 it is easy to see that the branching layout uses more cable types of a small capacity (that means more cheaper cables) and less cable meters in total. However, it might be more expensive since additional switch gears have to be included in the cost model. This strategy is rarely investigated but some researchers found out that allowing branching is worthwhile, cf. [3, 14].

In the course of this thesis we will present solutions for both options, compute their costs and compare them subsequently.



(a) Connection of nodes using strings only. (b) An example for allowing branching.

Figure 4: Two different approaches for computing the optimal layout. Cables with capacities k_i, $i \in \{1, 2, 3, 4, 5\}$, are being used.

As mentioned before, usually also the constraint of not allowing crossings of cabling is used. This has several reasons. One of them is that if crossings of cables are allowed one cable has to be buried underneath the other one. In case of failure of the lower placed cable and the need of replacing it both would have to be dug up which leads to significantly higher costs, cf. [1]. For similar reasons, we will proceed the same way.

Some of the approaches are very realistic since they include a lot of different features. For instance in [18] the dynamic seabed and bathymetry is considered.

We also want to represent reality as accurate as possible. The features that we include in our model are presented in the following sections.

1.4 Cabling in Solar Tower Power Plants

There are two types of cables used for transmitting data within the heliostat field. On the one hand a fiberglas cable is used to distribute the signal in the field. On the other hand a copper cable is used for the heliostats to receive the data. Next to the cabling price we have to consider costs for switches that have to be installed at every heliostat. Different versions lead to different costs. They are explained in detail in the following section.

In contrast for power cables switches are not needed and thus branching of power cables does not affect the costs. Different cable types made of copper having a different cross section are used to provide each heliostat with power.

Since the data and the power cable have different requirements we consider them as individual problems and optimize them separately. Nevertheless, some features affect both cable types as stated below.

As mentioned already in Subsection 1.3 an important feature we include in the model for both cable types is that we do not allow crossings of cables. All cables are buried one meter below ground level. In case of crossing the cables have to be laid one above the other which would lead to higher installation costs. Besides, when trenching and laying of cables follow immediately successive the lower cable could be damaged while digging the trench for the upper cable. That is why crossings should be avoided.

If quantity discounts rise with the purchase of a certain amount of a product, in our case cable meters, one refers to scaled prices. Since the tendency of the size of solar tower power plants is rising we neglect these prices in our model. These heliostat fields require that much cable meters that the threshold of the markdown is exceeded in any case.

Another feature that we do not consider is the topography of the field the solar tower is built in for computing the distances between heliostats. The power plants are mostly already chosen to be placed in flat areas and very small hills do not affect the results of our model gravely.

1.5 Modelling Approach

In order to implement the constraints and calculate the best layout we proceed stepwise and use different heuristics. Heuristics do not yield conclusively to the optimal solution but they provide a sufficient solution in an acceptable lapse of time. In each step we will modify the heuristic already used or change to another one to be able to consider the cost model and every single constraint of the respective step.

All algorithms are implemented in MATLAB. The visualization of the computed layouts is also done with the help of MATLAB. For distinguishing between the different cable types that are explained in detail in Sections 2.1 and 5.3 we use different colors.

In Section 2 the constraints and cost model for the data cable are presented in detail. The presentation of the algorithms used in our model and the optimization of the data cable can be found in Section 3. In the following Section 4 we compare the results and determine the most suitable algorithm for optimizing the data cable. Section 5 gives an overview of the power cable by listing the constraints and cost model. Section 6 deals with the optimization of the power cable followed by a comparison of results in Section 7. After examining the best algorithm for the power cable we perform a case study in Section 8 and apply the best algorithms for both data and power cable to the huge heliostat field. The thesis closes with a conclusion and outlook in Section 9.

2 Data Cable Model

In this section we want to outline the characteristics of the cables and technology used for transmitting data within the heliostat field. We then present the cost model and constraints that have to be considered to make the model as realistic as possible. Our goal is to compute an optimal layout for the data cables used in the heliostat field with minimal costs by considering the necessary constraints. We present different approaches solving the problems to be able to compare the quality of the different results.

2.1 Characteristics of the Data Cable

As mentioned before there are two types of cables used for transmitting data within the heliostat field: cables made of fiberglas and cables made of copper. The fiberglas cable has no length restriction and it is able to provide any number of heliostats with signal. However, there exist protocols that confine the attendance numbers of heliostats in each subnetwork so that we consider a maximum amount of heliostats per cable at some point in our model. In contrast the copper cable is only allowed to have a maximum length of $\ell_{\max} = 100$ meters and only connects two heliostats with one another.

Both the copper and the fiberglas cable are uniform so that we only distinguish between these two cable types.

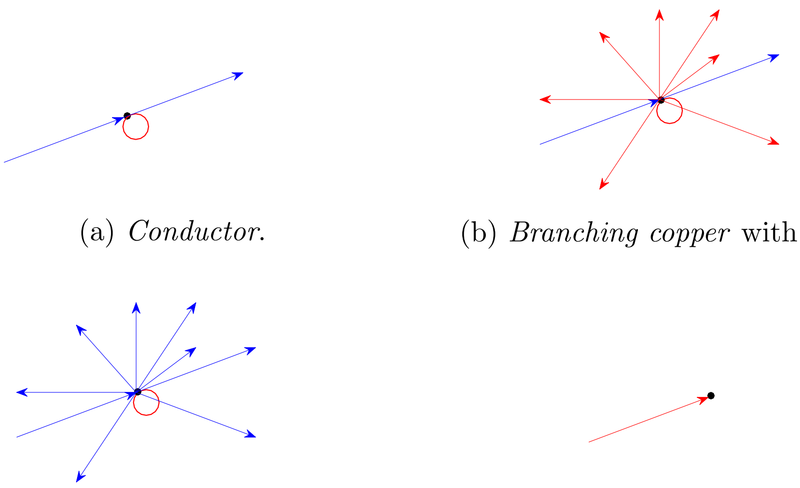
Each heliostat is equipped with a so called local control unit (LOC) to receive the data and allow branching. We distinguish between four different types of LOCs that are also presented visually in Figure 5.

1. *Conductor*
   Input: one fiberglas cable
   Output: one fiberglas cable and one copper cable (connected to the heliostat itself)

2. *Branching copper*
   Input: one fiberglas cable
   Output: one fiberglas cable and b_{maxCU} copper cables, $b_{\text{maxCU}} \in \mathbb{N}_{\geqslant 2}$ (from which one goes to the heliostat itself)

3. *Branching fiberglas*
   Input: one fiberglas cable
   Output: b_{maxFG} fiberglas cables and one copper cable, $b_{\text{maxCU}} \in \mathbb{N}_{\geqslant 2}$

4. *Endpoint*
   Input: one copper cable
   Output: none



(a) *Conductor*. (b) *Branching copper* with $b_{\text{maxCU}} = 8$.

(c) *Branching fiberglas* with $b_{\text{maxFG}} = 8$. (d) *Endpoint*.

Figure 5: The LOCs that are used for the data cable. Fiberglas cable is represented by blue and copper cable by red arrows.

In our model we allow an arbitrary number of LOCs of each type in the heliostat field.

Since the available switches for branching of fiberglas cable are costly we will compare using these switches with a solution that lays several cables into one trench and considers LOCs of type *conductor* only.

As solar tower power plants are mostly placed in sandy areas and are therefore exposed to sand storms, it is important for the heliostats to be able to move as quickly as possible into a horizontal position to protect themselves from damage. Moreover, in case of an overheating of the receiver which is mounted on top of the tower, the mirrors have to change into another position fast.

2.2 Cost Model

The total price for the laying of the cables in the heliostat field consists of the price of the cable material, protective foil and installation costs. The latter describes the costs for manpower that install the cables. As the salary differs from country to country we include this value as a parameter so that our model is even more adjustable for calculating the costs depending on the location of the planned solar tower power plant. For our test cases we chose Spain and South Africa; the installation costs can be seen in the following tables. Protective foil is placed above the cables to let workers know where cables are laid to prevent them from digging further. On the one hand cables could be damaged and on the other hand workers could injure themselves or even die in case of the cables being power cables.

Materials affecting the total costs		Price	Parameter
Cable	Fiber optic cable	2 €/m	c_{cableFG}
	Ethernet cable	0.7 €/m	c_{cableCU}
Additional material	Protective foil	2 €/m	c_{foil}
Manpower/ digging	Spain	25 €/m^3	c_{manpower}
	South Africa	5 €/m^3	

Table 1: Costs for cables and foil.

We proceed on the assumption that the trenches being dug have a depth and a width of 1 m. Therefore, the total costs of laying 1 m fiberglas cable into a 1 m^3 trench are calculated by

$$c_{\text{totalFG}} = c_{\text{cableFG}} + c_{\text{foil}} + c_{\text{manpower}}$$

and for copper cable respectively

$$c_{\text{totalCU}} = c_{\text{cableCU}} + c_{\text{foil}} + c_{\text{manpower}}.$$

The following table shows the total costs of installing 1 m fiberglas and 1 m copper cable respectively dependent on the country the solar tower power plant is built in.

Installation with	Spain	South Africa	Parameter
Fiber optic cable [€/m]	29	9	c_{totalFG}
Ethernet cable [€/m]	27.7	7.7	c_{totalCU}

Table 2: Total costs of laying of cables with respect to the country the tower is built in, cf. [8].

Besides that, the following costs arise for the switches used to connect copper and fiberglas cables and for connecting fiberglas cables only:

LOC	Price [€/piece]	Parameter
Endpoint	10	c_{endpoint}
Conductor	100	$c_{\text{conductor}}$
Branching copper ($b_{\text{maxCU}} = 8$)	500	$c_{\text{branchingCU}}$
Branching fiberglas ($b_{\text{maxFG}} = 8$)	500	$c_{\text{branchingFG}}$

Table 3: Costs of LOCs.

Consider that normal switches only work at temperatures up to 50°C hence they are not suitable to be installed in solar fields. Therefore the use of switches that are constructed for a surrounding temperature up to 80°C such as the *Moxa T-series* are recommended.

An overview of the additional parameters used in our model is listed below.

	Value	**Parameter**
Length restriction copper cable	100 m	$\ell_{\max}$
Amount of heliostats per subnetwork dependent on the used protocol	128 pieces	$p_{\max}$
Amount of outputs of fiberglas cable at LOC of type *branching fiberglas*	8 outputs	$b_{\max FG}$
Amount of outputs of copper cable at LOC of type *branching copper*	8 outputs	$b_{\max CU}$

Table 4: Additional parameters that are used in the model.

For convenience we will use the term fiberglas cable instead of fiber optic cable and copper instead of Ethernet in the following chapters.

In order to calculate the optimal layout we will proceed stepwise considering more and more constraints. The cost model and the constraints as well as the method to solve the respective problem are presented in Table 5. The method named in each row always considers the constraints and cost model of the same row and the ones above.

Method	**Constraints**	**Cost model**
1: Hamiltonian path	• Each heliostat has to be connected to the solar tower • Each heliostat shall only be connected once	• Cable meters used to connect the heliostats • Price of the cable • Installation costs to connect the heliostats with the solar tower • Consideration of LOC of type *conductor*
2: Hamiltonian path with 2-opt heuristic	• Cables are not allowed to cross	
3: s-Hamiltonian path with 2-opt heuristic	• Consider the used protocol that confines the attendance numbers of heliostats in each subnetwork	
4: Kruskal's algorithm	• The cable is allowed to branch at the heliostats	• Including switches for branching of the fiberglas cable
5: Esau Williams heuristic	• The distance between two heliostats connected by a copper cable has to be less than 100 meters	• Distinction between fiberglas and copper cable • Additionally consideration of LOCs of type *endpoint* and *branching copper*

Table 5: Overview of the constraints and cost model. We will proceed in five steps. The method for each step is listed in the left column.

3 Optimization of the Data Cable

In this section we present the different steps as explained in Subsection 2.2 for computing the optimal layout of the data cable. Depending on which country the power plant is built in one of the implemented algorithms might be more suitable than others. Since we will interpret the heliostat field as a graph, we first present a few definitions and terms that are used for explaining the heuristics and algorithms in this section.

Note that in this and in the sixth section we will figure the computed layouts for Spain only. The results for South Africa are presented in bar charts in Sections 4 and 7.

3.1 Excursion to Graph Theory

Let V $(= Vertices)$ and E $(= Edges)$ be two disjoint and non-empty sets with $v_1, ..., v_n \in V$ and $e_1, ..., e_m \in E \subset V \times V$. Further we define $P(V) := \{X \subseteq V \mid 1 \leqslant |X| \leqslant 2\}$ where $|X|$ describes the cardinal number of X. If $g : E \to P(V)$ is a mapping, the ordered pair $G = (V, E, g) = (V(G), E(G))$ is called a **graph**.

- If $e \in E$ with $g(e) = \{v, w\}, v, w \in V$ and $v = w$, then e is a **loop**.
- We denote two different edges $e_1, e_2 \in E$ with $g(e_1) = g(e_2) = \{v, w\}$ and $v, w \in V$ as **multiple edges**.
- A graph G is **simple** if it does not have any loops and multiple edges.
- G is called **complete** if it is simple and undirected and $\forall v_1, v_2 \in V$ exist $g(e) = \{v_1, v_2\}$ where $e \in E$.
- Let $G = (V, E, g)$ be a graph and $e_1, ..., e_p \in E$ with $g(e_i) = \{v_{i-1}, v_i\}$ for $i = 1, ..., p$. The sequence $P = v_0 e_1 v_1 e_2 ... e_p v_p$ is called a **path** if all vertices and all edges in P are pairwise disjoint.
- If G holds a path P with $V(P) = V(G)$, we call P a **Hamiltonian path**.
- Two edges $v_1, v_2 \in V(G)$ are **connected**, if a path from v_1 to v_2 exists. G is called connected if v_1 and v_2 are connected $\forall\ v_1, v_2 \in V(G)$.

3.2 Hamiltonian Path made of Fiberglas

As a first approach we use cables made of fiberglas only and do not allow any branching in our heliostat field because initially we only consider the LOC of type *conductor* as stated in Table 5. To compute a first valid solution we proceed in three steps that are presented below.

3.2.1 Step 1: Connecting each Heliostat

We are looking for a layout that connects each heliostat once while considering costs for cable. For this purpose we calculate all distances from one heliostat to another and then starting at the solar tower we connect the heliostat with the shortest distance to the tower. In each further step the algorithm searches for the heliostat having the shortest distance to the most recently connected one and connects them with a cable if this heliostat has not been visited yet. As we only use one cable we create a Hamiltonian path.

The described algorithm is also known as **Nearest Neighbor Heuristic**.

We define the heliostats and the solar tower as nodes, the edges and their weights represent the distances between them.

The distance between two heliostats H_i and H_j with the coordinates (x_i, y_i) and (x_j, y_j) is calculated by the Euclidean distance for two dimensions which is equivalent to the Pythagorean theorem:

$$dist_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}.$$

The nearest neighbor heuristic considers $n-1$ times the smallest distance to the next node that has not been visited yet, that is a maximum of $n-1$ distances in total. This results in a runtime of $\mathcal{O}(n^2)$, cf. [11].

Since we do not distinguish between different LOCs yet each heliostat is equipped with a LOC of type *conductor*. As we can see in Figure 6 this solution is only suitable to a limited extent. There are plenty of crossings and some connections are very long because the algorithm only searches for a local optimum. These edges could be replaced easily with shorter alternatives by allowing branching or reorganizing them. Since we are interested in a valid solution we revise the current layout in the next step.

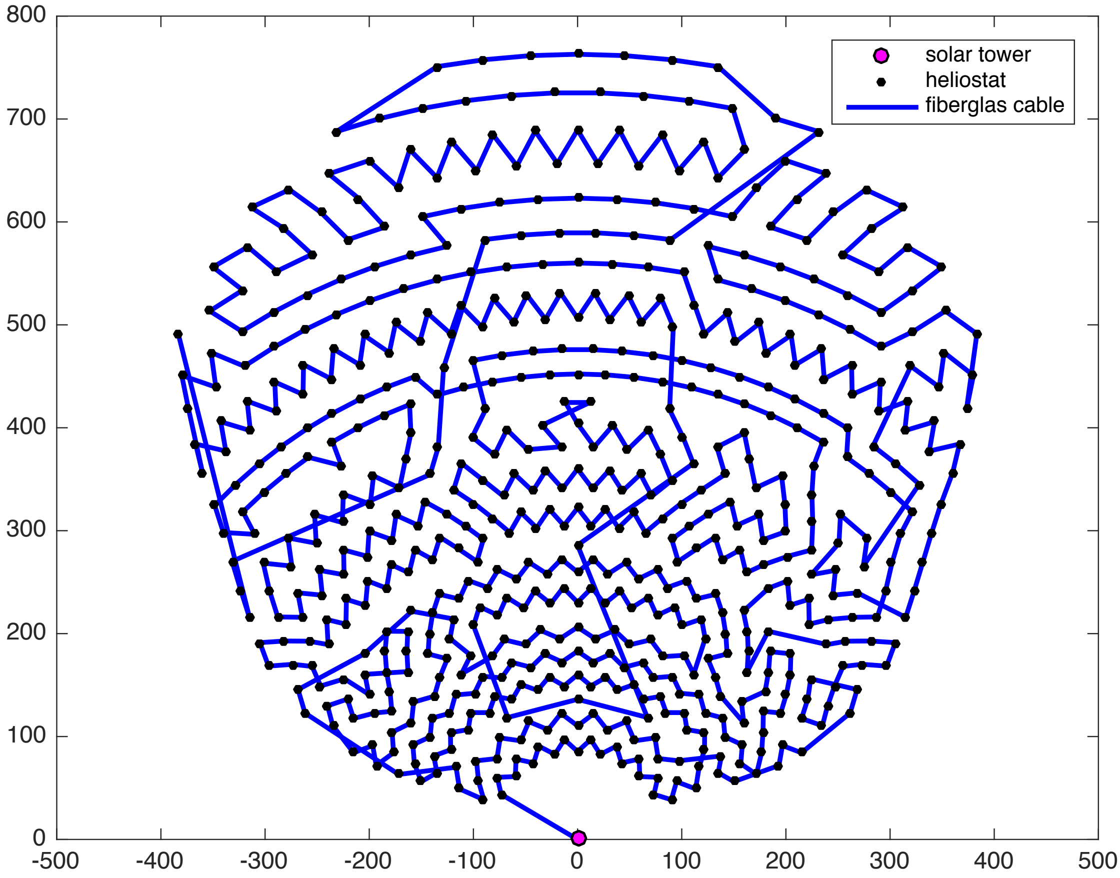


Figure 6: Solar tower: PS10, country: Spain, fiberglas cable length: 18 421.19m, total costs: 596 614.63€. Hamiltonian path computed with the nearest neighbor algorithm. Several crossings as well as long connections occur.

3.2.2 Step 2: Eliminate Crossings

Since the previous solution is invalid because of cable crossings we modify the layout in the next step. In order to get a Hamiltonian path without any crossings we chose to apply the 2-opt algorithm, cf. [5]. It is a heuristic that is used to improve an already generated solution, mostly for the travelling salesman problem (TSP).

This problem is about finding the cheapest route a travelling salesman can take if he or she wants to visit a fixed number of nodes, each of them exactly once, and eventually return to the starting point.

Even if we created a Hamiltonian path instead of a cycle we can still apply the 2-opt heuristic simply by adding one edge that connects the solar tower with the heliostat being at the other end of the path. In the end we remove the longer edge of the two that are connected with the solar tower to get a valid solution.

2-opt heuristic: As explained in [5] the algorithm starts with an initial tour and improves it stepwise by selecting two edges e_1 and e_2 with $g(e_1) = \{v_1, w_1\}$ and $g(e_2) = \{v_2, w_2\}$ from the tour and replaces it with $g(e_{new1}) = \{v_1, v_2\}$ and $g(e_{new2}) = \{w_1, w_2\}$ if and only if the length of the edges e_{new1} and e_{new2} is smaller than the length of e_1 and e_2. The algorithm terminates when the length of the tour cannot be shortened anymore. The result is a local optimum.

As can be seen in Figure 7 the 2-opt algorithm exchanges crossings for direct connections since they are shorter in the two-dimensional Euclidean plane.

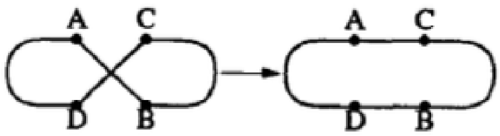


Figure 7: An exemplary swap of the 2-opt algorithm, cf. [2].

As stated in [21] the runtime of 2-opt is no more than $\mathcal{O}(n^3)$. For computing the solution for PS10 using Matlab only 2.9 seconds are needed. The final result can be seen in Figure 8. Compared to the previous solution about 1 700 meters of cables are saved.

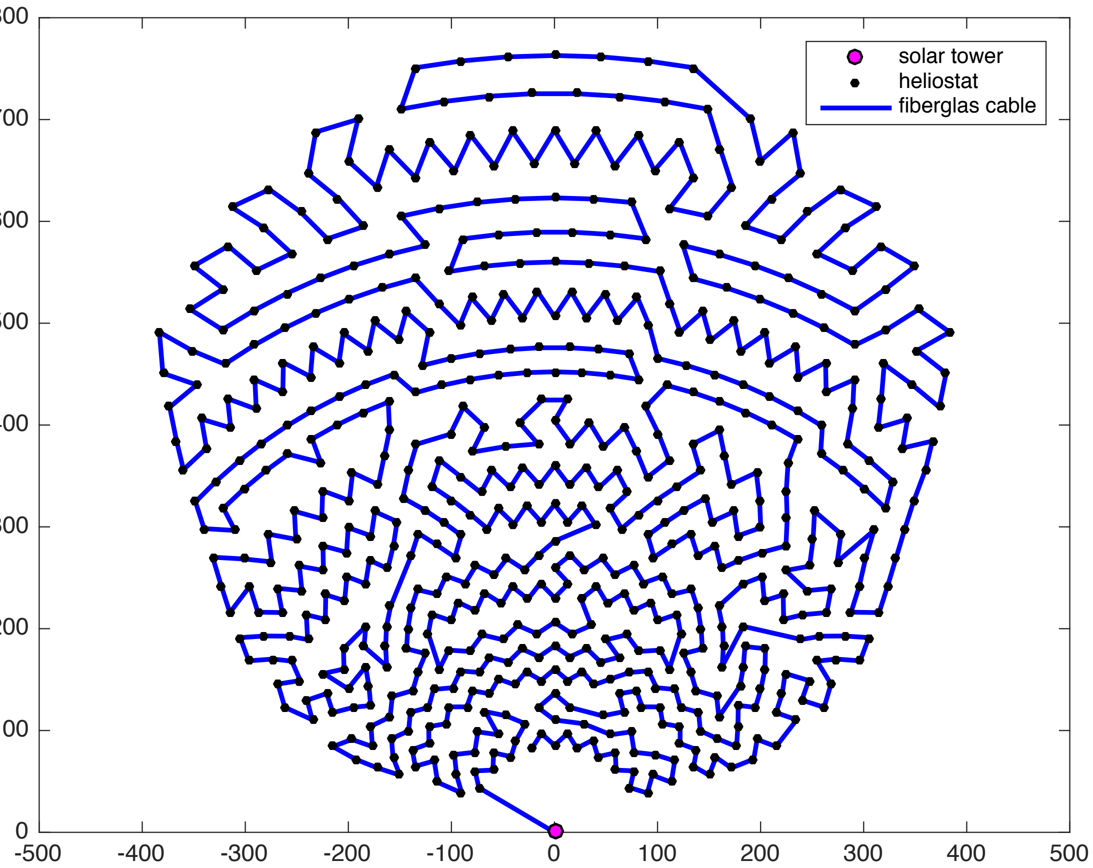


Figure 8: Solar tower: PS10, country: Spain, fiberglas cable length: 16 747.28m, total costs: 548 071.23€. Layout after applying the 2-opt heuristic. All crossings are removed.

3.2.3 Step 3: Limit the Attendance of Heliostats per Subnetwork

As already mentioned in Section 2 some protocols only allow a limited amount of heliostats that are connected to a wiring loom. For instance the *Shagaya project* in Kuwait that was among others planned by the company *TSK Flagsol* uses a protocol that confines each subnetwork to 128 heliostats connected in total. We want to add this feature in our model and run our test case with at most $p_{\max} = 128$ heliostats per subtree. Of course this value can be changed by the user individually. With PS10 having $h_{\text{PS10}} = 624$ heliostats we get

$$s = \lceil \frac{h_{\text{PS10}}}{p_{\max}} \rceil = \lceil \frac{624}{128} \rceil = \lceil 4.897 \rceil = 5.$$

Hence s is the amount of subnetworks that are required in the field.

While we created a single Hamiltonian path only in the previous subsection it now is a s-Hamiltonian path problem that has to be solved.

We decided to divide the heliostat field into s *cake pieces* each containing at most p_{max} heliostats and create a 2-optimal Hamiltonian path for each section.

In contrast to the solution presented in Subsection 3.2.2 the length of cable meters raises by approximately 830 meters. Among others this results from $s = 5$ cables starting from the solar tower instead of only one. The layout can be seen in Figure 9. The runtime for this example is 2.5 seconds.

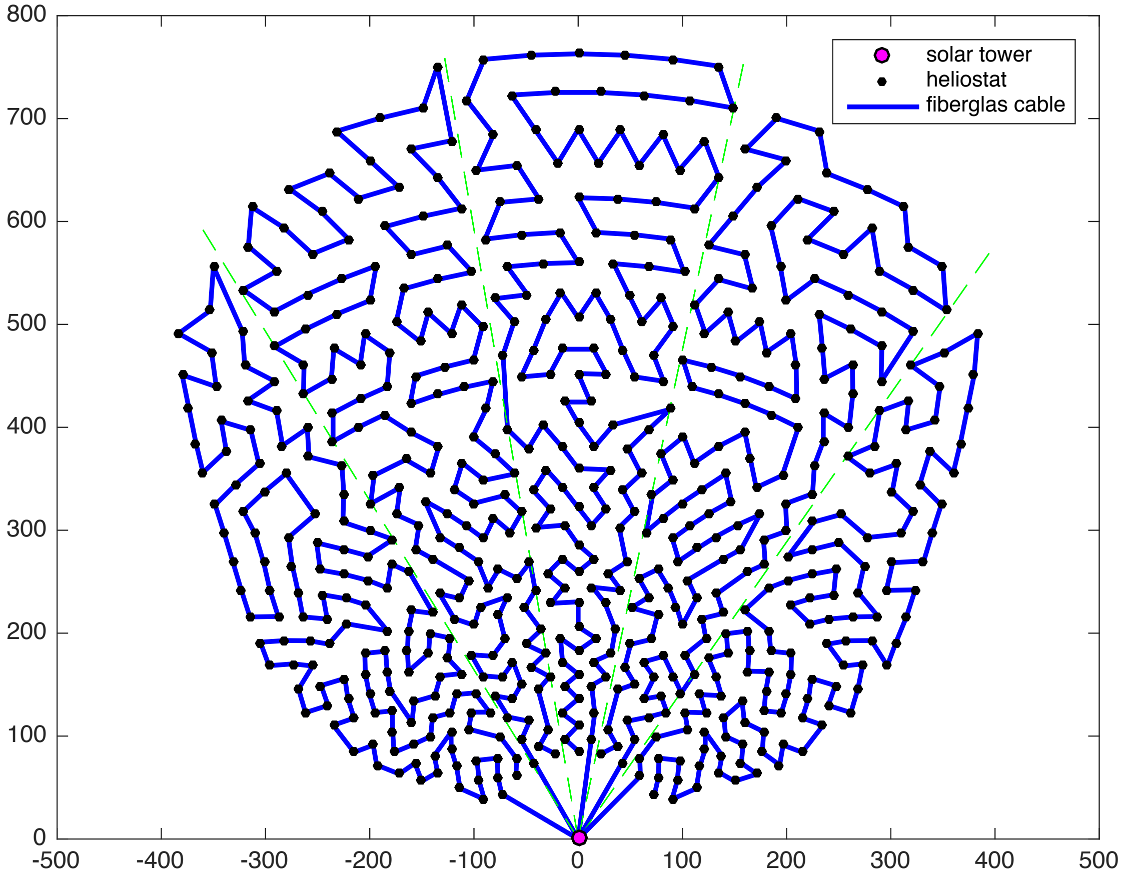


Figure 9: Solar tower: PS10, country: Spain, fiberglas cable length: 17 583.33m, total costs: 572 316.59€. Five Hamiltonian paths made of fiberglas cable starting from the solar tower.

3.3 Step 4: Allowing Branching and Consideration of Switches for Fiberglas Cable

The object of this subsection is to minimize the cable meters while allowing branching. Next to the LOC of type *conductor* we now make use of the LOC of type *branching fiberglas* as well. We will present two algorithms. The first one is rather naive while the second one is based on Kruskal's algorithm that creates a tree with minimal length.

3.3.1 Naive Circular Pattern

Before considering both fiberglas and copper cables we present a naive approach meaning the cables being installed along circles around the solar tower which can be seen in Figure 10. Since a maximum of $p_{\max} = 128$ heliostats is allowed per subtree $s = 5$ cables start from the solar tower and are laid in the trench running on the right side of the heliostat field. Each cable connects several semi-circles. Thus mainly LOCs of type *conductor* and only few LOCs that allow branching of the fiberglas cable are being used.

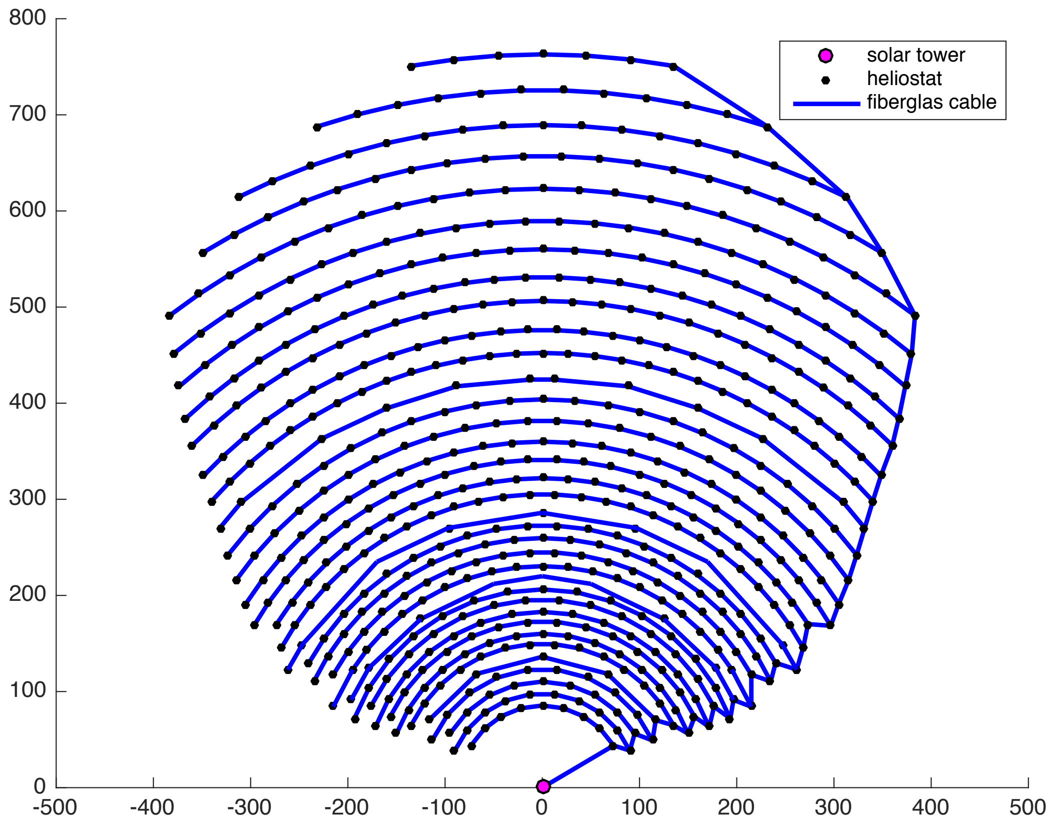


Figure 10: Solar tower: PS10, country: Spain, fiberglas cable length: 23 671.66m, trench length: 21 038.34m, total costs: 689 378.44€. Naive data cable layout of PS10 with the cables being installed in semi-circles.

In contrast to the $s-$Hamiltonian path both the cable length and the total costs are significantly higher when applying the naive circular pattern. However, this approach will be of importance at a later date when computing the optimal layout for the DLR Field.

3.3.2 Minimum Spanning Tree

We will compute a layout with minimal cable length and minimal cable costs by creating a **minimum spanning tree** (MST). Furthermore, our aim is to compare the option of using switches for fiberglas cable with the option to lay several cables into one trench and do not allow branching. In the end we will see which one is the more economic and thus the recommended procedure.

A minimum spanning tree is a subgraph T of a connected graph G that has weighted edges, in our case the Euclidean distances between the heliostats. T contains a minimum weight set of edges of G that connects all nodes, cf. [19].

Kruskal's algorithm:

Input:

- A connected and simple graph $G = (V, E)$, $|V| = n \geqslant 3$, $|E| = m$
- Edge-weighted function $c : E \to (0, \infty)$

Output:

- A minimum spanning tree of G

Algorithm:

- Sort the edges of G by weights $c(e_1) \leqslant c(e_2) \leqslant ... \leqslant c(e_m)$.
- $S := \emptyset$.
- For $i = 1, ..., m$ check if $(V, S \cup \{e_i\})$ contains a cycle
  - if yes: $S := S$
  - if no: $S := S \cup \{e_i\}$.
- Return (V, S) and stop.

Kruskal's algorithm has a runtime of $\mathcal{O}(m \cdot log(m))$.

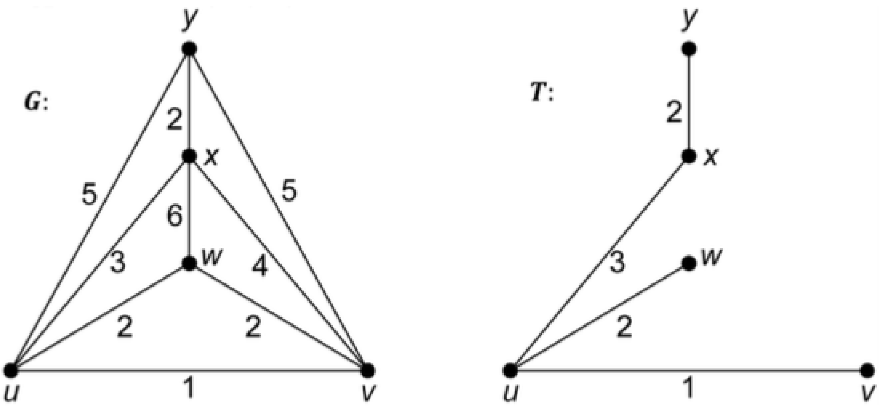


Figure 11: An exemplary graph G and its minimum spanning tree T computed with Kruskal's algorithm, cf. [9].

Just like in Subsection 3.2.3 we divide the field into s cake pieces first and apply Kruskal's algorithm to each area to satisfy the restriction of the used protocol. The runtime for this example is 2.6 seconds. The result can be seen in Figure 12.

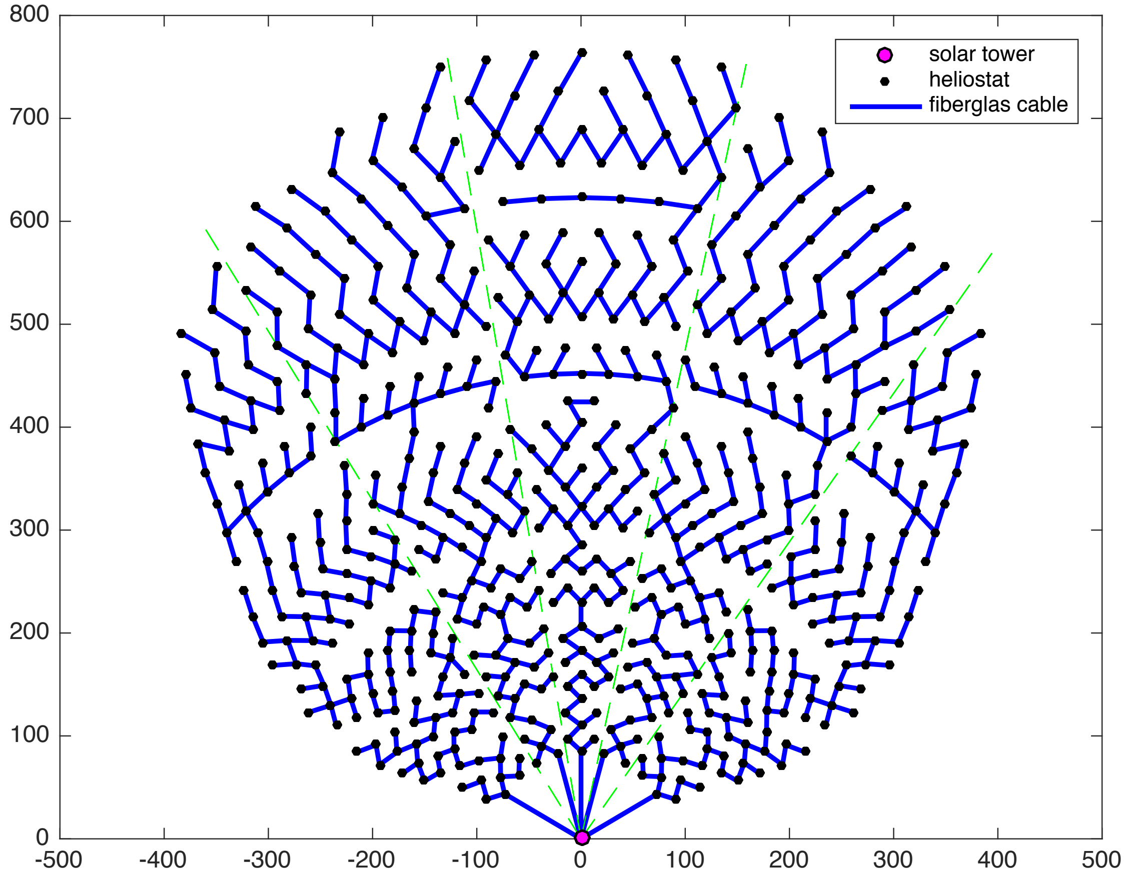
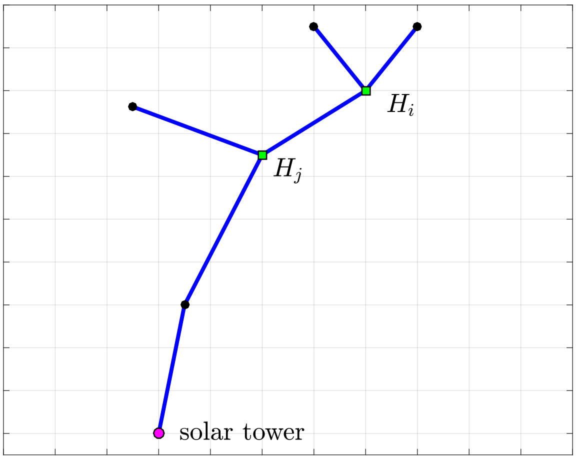
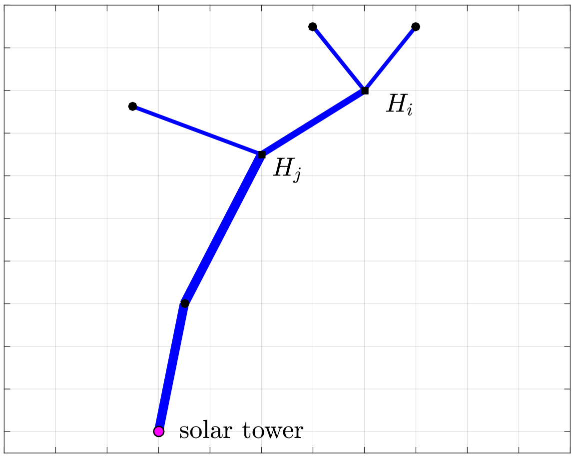


Figure 12: Solar tower: PS10, country: Spain, fiberglas cable length: 16 392.27m, total costs: 596 575.82€. Five minimum spanning trees that connect the heliostats in PS10.

To compute the total cabling costs of the minimum spanning tree the total length of its edges is multiplied with the total costs of the fiberglas cable as mentioned in Subsection 2.2. Compared to the length of the s-Hamiltonian path approximately another 1 200 meters of cabling can be saved when allowing branching. However, since we allow branching of the fiberglas cable the total price raises because we make use of the expensive switches of type *branching fiberglas*. The computed layout contains 147 branching points and thus leads to total costs of 596 575.82€.



(a) Option 1: A switch of type *branching fiberglas* is installed at heliostats H_i and H_j (green square).



(b) Option 2: Cables are installed parallel (shown by thicker lines).

Figure 13: Two possibilities to install cables when allowing branching. One square in the grid has a side length of 20 meters.

Another possibility is to lay more than one cable into one trench and spare the fiberglas switches for branching and use the ones of type *conductor* instead. This is only worthwhile if the length of the cables that would be laid additionally into one trench exceeds

$$\sum_{i \in I} \frac{c_{\text{branchingFG}} - c_{\text{conductor}}}{c_{\text{cableFG}}},$$

where $I := \{v_i \in V : v_i \text{ has more than one successor}\}$.

An example is shown in Figure 13. The total price for option 1 (Figure 13a) consists of the total cabling price and the costs for two switches of type *branching fiberglas* and four of type *conductor*. In option 2 (Figure 13b) three cables are laid in the trench from the solar tower to heliostat H_j and two cables in the trench from H_j to H_i. Therefore, the total costs are composed of the costs for laying one cable in the trenches, six switches of type *conductor* plus additionally the costs for cabling from the tower to H_j and to H_i.

The additional cable meters that have to be laid into the trenches in option 2 are calculated by

$$\underbrace{2 \cdot (\sqrt{30^2 + 70^2} + \sqrt{10^2 + 60^2})}_{\text{two additional cables from the solar tower to } H_j} + \underbrace{\sqrt{40^2 + 30^2}}_{\text{one additional cable from } H_i \text{ to } H_j} \approx 323.97 \text{ meters.}$$

Since $I := \{H_i, H_j\}$ and thus

$$\sum_{i \in I} \frac{c_{\text{branchingFG}} - c_{\text{conductor}}}{c_{\text{cableFG}}} = 2 \cdot \frac{500 - 100}{2} = 400 \text{ meters}$$

it would be more economic in this example to lay several cables into one trench instead of installing switches of type *branching fiberglas*.

In the computed layout of PS10 the length of the additional cable meters must not exceed

$$\sum_{i \in I} \frac{c_{\text{branchingFG}} - c_{\text{conductor}}}{c_{\text{cableFG}}} = 147 \cdot \frac{500 - 100}{2} = 29\,400 \text{ meters}$$

to make installing cables parallel more economic than using LOCs of type *branching fiberglas*. With 106 080.57 meters of cabling that has to be laid into the dug trenches additionally this value is clearly exceeded and the total costs add up to 749 936.97€. Thus using switches for branching is highly recommended. Laying more than one cable into one trench and spare LOCs of type *branching fiberglas* is efficient on a small area only. We will get back to this in Subsection 3.4.3. Despite that both options are more expensive than the s-Hamiltonian path so that we continue with the next step and leave this one as a possible but poor solution even if the cable length is the lowest of all previous steps.

3.4 Step 5: Combination of Fiberglas and Copper Cables

Up to now we only considered cable made of fiberglas in our model. Since the cable price per meter is approximately three times higher than the price for one meter of copper cable we want to generate a solution that considers both types of cable.

A solution with the use of copper cables only would rise the total costs significantly because thicker cables especially near the solar tower have to be chosen to guarantee that even the heliostats placed further away still get a data signal. Therefore, we neglect this option and generate a solution that combines both the fiberglas and the copper cables instead.

3.4.1 Esau Williams Heuristic

Since the used copper cable is able to provide only one heliostat with data and due to the used protocol limiting the capacity of each subnetwork we can trace back the problem of computing an optimal cable layout to creating a **multilevel capacitated minimum spanning tree** (MLCMST).

Consider a simple and complete graph $G = (V, E)$ with a positive edge-weighted function $c : E \to (0, \infty)$, a root node $r \in V$ (solar tower) and $k \in \mathbb{N}$. In our case c assigns to each connection between two heliostats the capacity one to consider the cable capacities. A **capacitated minimum spanning tree** (CMST) is a minimum cost spanning tree that fulfills the constraint of each subtree being connected to r having a total vertex-weight of no more than k, cf. [12].

According to [12] "the most popular and efficient algorithm for the CMST problem is due to Esau and Williams". It has a runtime of at most $\mathcal{O}(n^2 \cdot log(n))$ and is therefore chosen to be implemented.

Esau Williams heuristic:

Input:

- A connected and simple graph $G = (V, E)$, $|V| = n \geqslant 3$, $|E| = m$
- Edge-weighted function $c : E \to (0, \infty)$
- Root node $r \in V$
- Capacity restriction $k \in \mathbb{N}$ for each subtree

Output:

- A minimum spanning tree of G, each subtree observing the restriction k

Algorithm:

1. Connect each vertex $v \in V$ to the root node r.
2. For each $v_i \in V$ that is directly connected to r and for each $v_j \in V$ that is not connected to r via v_i compute

$$savings_{ij} = c_{ij} - c_{ir},$$

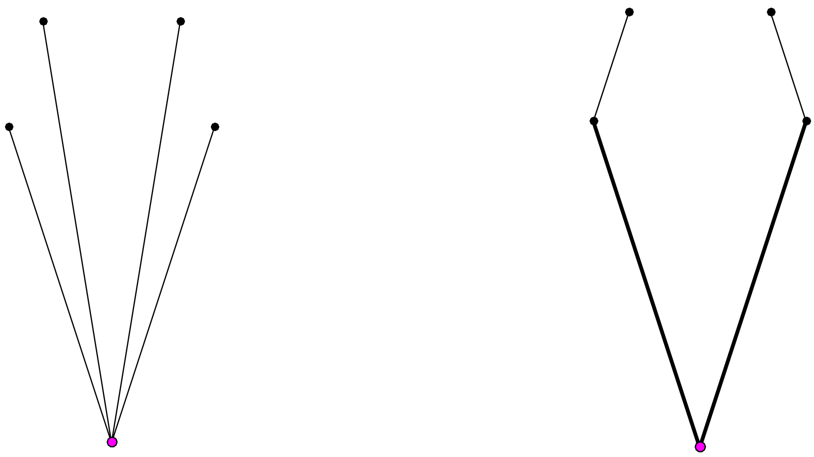
   where c_{ij} are the total costs for the cabling from heliostat H_i to H_j. Take account of the additional costs that occur when another LOC is needed.
3. Choose v_j with the maximum savings (most negative value). If the capacity k is not exceeded by the sum of vertex weights of both subtrees connect v_i with v_j and delete the connection between v_i and r.
4. Repeat steps 2 and 3 until the lowest savings are positive.

The MLCMST is an expansion of the CMST with two or more different cable types given. These cables differ in their maximum capacity k_i and costs per meter c_i. As explained before the copper cable has a capacity of $k_1 = 1$ and the fiberglas cable of

$k_2 = protocol_{max}$ (= 128 in our test case) to fulfill the restrictions of the used protocol.

The Esau Williams heuristic can be modified for the multilevel capacitated minimum spanning tree problem. If the capacity of cable i is exceeded change to a cable j having a higher capacity and consider the additional costs that occur. This version has already been used in [14] for computing the optimal cabling in offshore wind farms. It will be the foundation of our approach computing the optimal cable layout in solar tower power plants.

Cables in offshore wind parks are not allowed to cross as well and the used simulation already includes this feature. Detailed explanation of the implementation avoiding crossings can be found in [14].



(a) At first each node is directly connected to the root node. (b) Result after applying the Esau Williams heuristic.

Figure 14: An example of creating a MLCMST with the help of the Esau Williams algorithm with capacities $k_1 = 1$ and $k_2 = 2$.

3.4.2 Step 5a: Consideration of all Types of LOCs

To cope with all constraints and requirements we modify the Esau Williams heuristic used in [14] to Esau Williams Version 1. In this step we include the four different LOCs that are described in detail in Subsection 2.1 in our model. We consider the maximum length of $\ell_{\max} = 100$ meters of the copper cables and choose the LOCs of type *branching* to have at most $b_{\text{maxCU}} = 8$ copper and $b_{\text{maxFG}} = 8$ fiberglas respectively cable outputs. Of course this parameter can be changed individually by the user and the following explanations will be valid for any $b_{\text{maxCU}}, b_{\text{maxFG}} \in \mathbb{N}_{\geqslant 2}$. The algorithm generates a MLCMST with cable capacities $k_1 = 1$ (copper cable) and $k_2 = p_{\max} = 128$ (fiberglas cable).

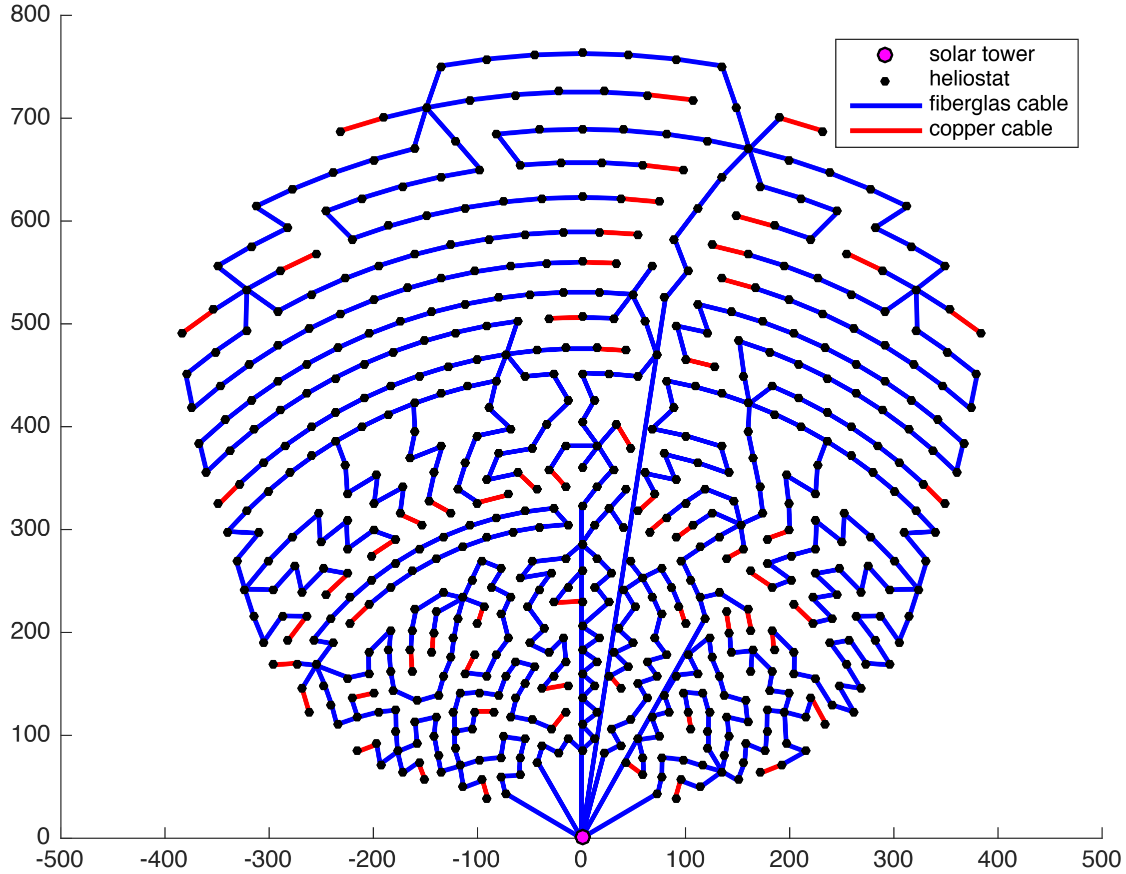


Figure 15: Solar tower: PS10, country: Spain, total cable length: 18 406.24m, total costs: 623 898.38€. Result of Esau Williams Version 1. Blue connections represent cables made of fiberglas, red cables are the ones made of copper.

As can be seen in Figure 15 a lot of LOCs of type *branching* are included in the computed layout. However, not all of them are connected with the maximum amount of eight heliostats but in most cases much less. This leads to a high amount of LOCs of this nature and therefore to higher costs since it is the most expensive LOC.

The total costs can be reduced by always connecting eight heliostats to each branching point so that less LOCs of type *branching* and more of type *endpoint* are needed.

Another conspicuousness is that there are three connections being several hundred meters long spanning large areas of the heliostat field. At first sight these cables seem to cross with shorter connections but taking a closer look shows that these cables touch the heliostats only. However, this is not a good solution since these long connections could be avoided by reorganizing some edges.

The computer we tested all previous algorithms on did not finish in less than 10 hours for the layout of PS10 so that we changed to a better one being equipped with a higher primary memory and more cores. Nonetheless, the runtime on the better computer still amounted to approximately 2 hours and 15 minutes so that we decided to reduce the problem to four subproblems in the final step. In this way we could handle all restrictions and assumptions easier and save a lot of runtime as can be seen in the next subsection.

3.4.3 Step 5b: Forming Groups

The final step is to make sure that almost every LOC of type *branching* is connected to the full amount of heliostats that we chose as output. We will only consider the branching switch for copper and neglect the one for fiberglas in this approach. The four subproblems as mentioned before are now presented in detail.

Subproblem 1: Divide the heliostat field into groups

In order to connect as many LOCs of type *branching copper* with $b_{\text{maxCU}} = 8$ heliostats we divide the heliostat field into groups mainly containing eight heliostats each with a few exceptions as will be explained later. This guarantees that as few branching points and as much endpoints as possible are used in the layout and therefore a lot of costs for the LOCs are saved as well as for the cabling because thus the use of fiberglas cable is minimized.

To create groups the algorithm divides the heliostat field into belts first. It pays attention to the thickness of each belt bearing relation to the radius of the later computed groups. For this purpose the groups in the current belt are counted. Since it is not guaranteed that the amount of heliostats in each belt is divisible by $b_{\text{maxCU}} = 8$, there may be one group that includes less than $b_{\text{maxCU}} = 8$ heliostats. The approximate width of each group – computed by dividing the length of the upper arc that edges the belt by the amount of groups – is supposed to have the same length as the thickness of the belt. The algorithm allows a deviation of 20% downwards and upwards. Afterwards, the center heliostat of each group is set. In doing so it is guaranteed that the groups are having the form of a circle rather than an ellipsis. This will largely prevent connections between the center and the other heliostats of the same group being greater than $\ell_{\text{max}} = 100$ meters.

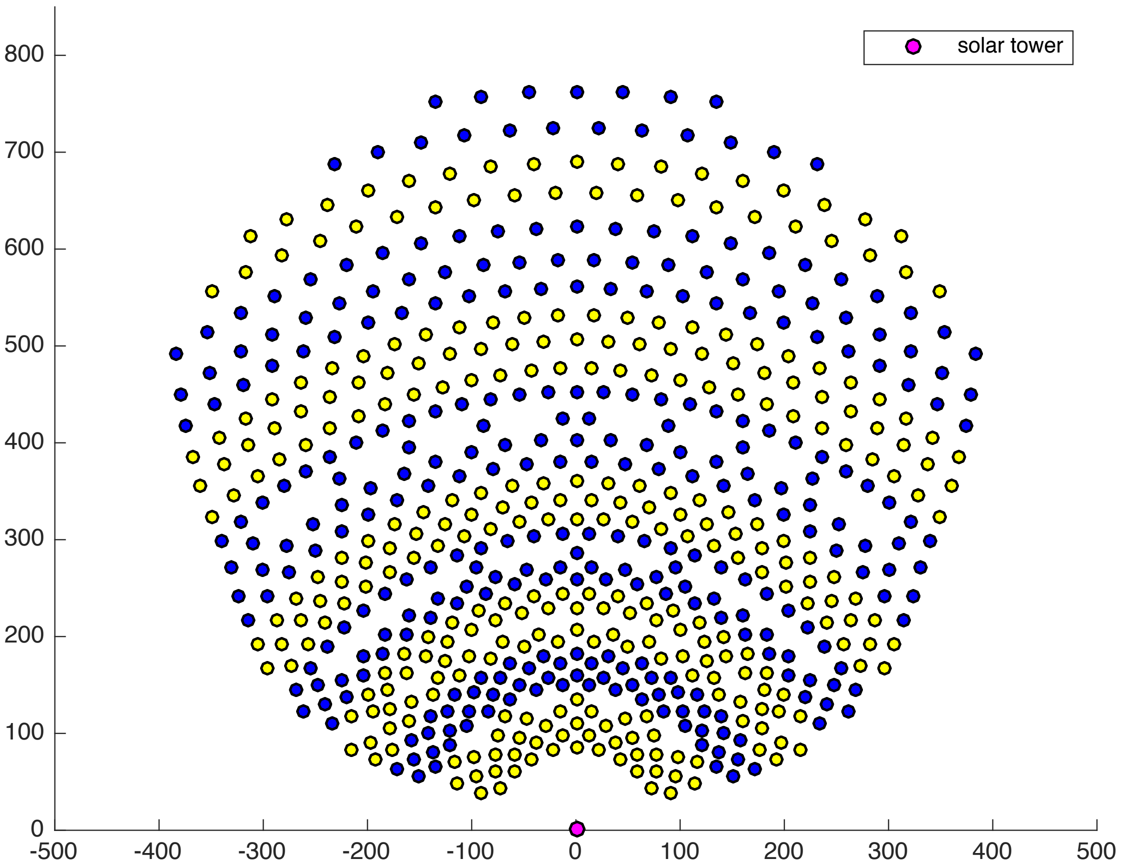
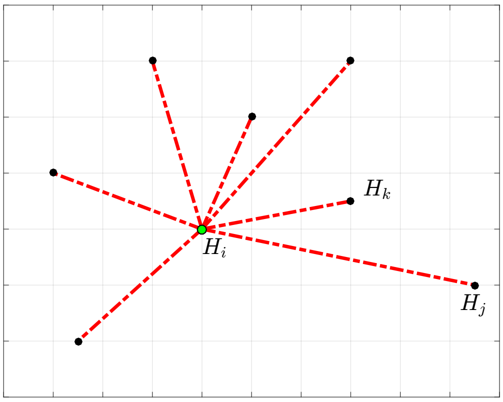


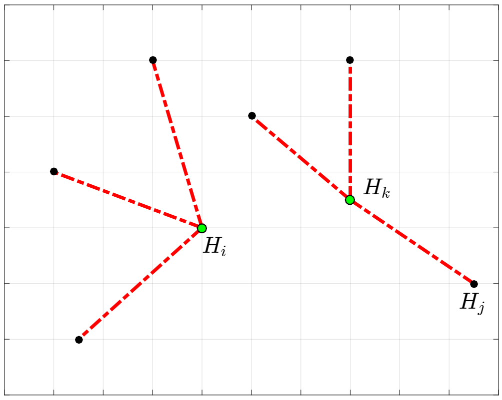
Figure 16: Solar tower: PS10. In a first step the algorithm creates belts. The corresponding heliostats are visualized in yellow and blue.

The algorithm checks if connections will occur that are longer than $\ell_{\max} = 100$ meters. If so the group is divided into two small groups as exemplary shown in Figure 17. Assume that the distance from heliostat H_i to heliostat H_j is longer than $\ell_{\max} = 100$ meters. In this case the algorithm creates another group and thus eliminates the prohibited connection. This rises the total costs since now two LOCs of type *branching copper* are needed to be installed for heliostats H_i and H_j. However, it is inevitable when creating a valid solution.

The centers of the groups of $b_{\text{maxCU}} = 8$ of PS10 are displayed in Figure 18.



(a) The distance from heliostat H_i to heliostat H_j exceeds $\ell_{\max} = 100$ meters.



(b) Revised and permissible layout. No connection is longer than $\ell_{\max} = 100$ meters.

Figure 17: Example for managing distances greater than $\ell_{\max} = 100$ meters. The copper cables that are laid in the final step are indicated in dashed red lines. One square in the grid has a side length of 20 meters.

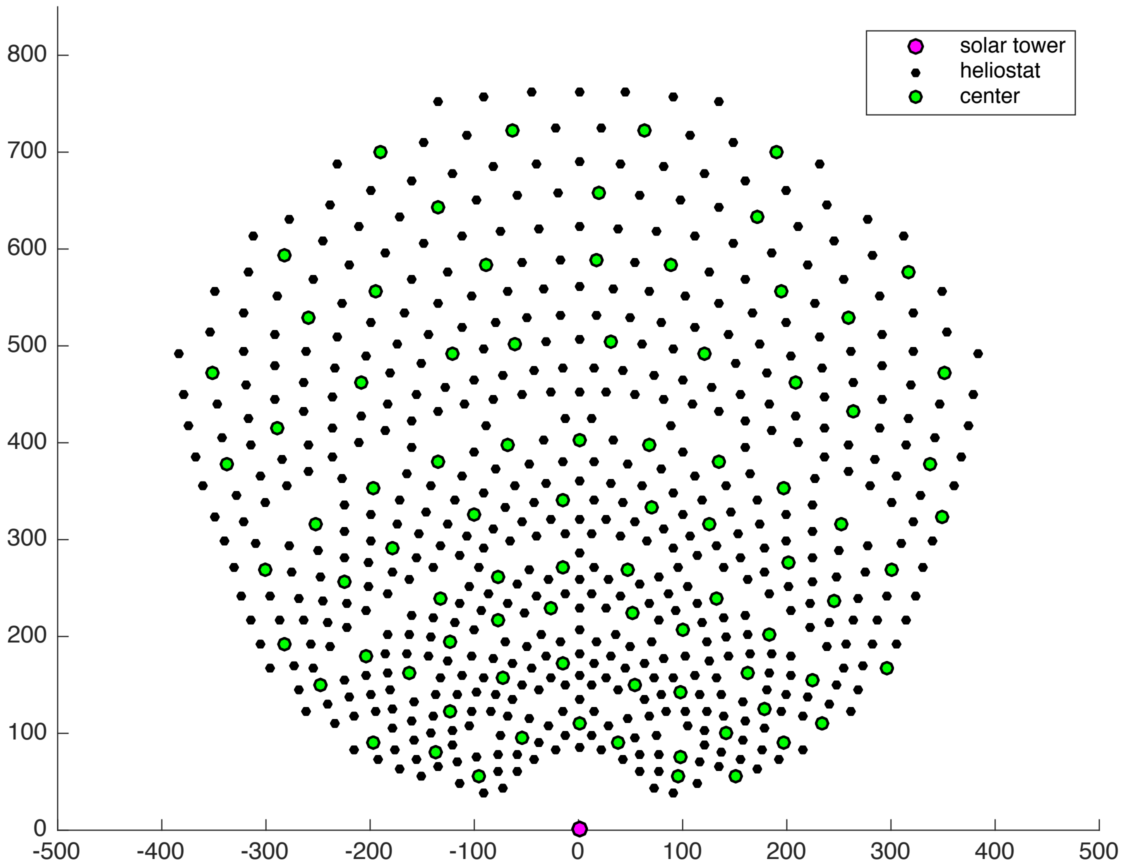


Figure 18: Solar tower: PS10. The centers of the groups of $b_{\mathrm{maxCU}} = 8$ are highlighted in green.

Subproblem 2: Performing Esau Williams Version 2 with the centers of each group of $b_{\mathbf{maxCU}} = 8$

After computing the centers of the groups of $b_{\text{maxCU}} = 8$ that will be the heliostats equipped with the LOC of type *branching copper* we perform the Esau Williams algorithm. Since we only consider the heliostats that will have this LOC installed we modify Esau Williams Version 1 to Version 2 and allow cables made of fiberglas only.

To take the protocol into account we only allow k centers to be connected by a cable. From Table 4 we have a maximum of $p_{\text{max}} = 128$ heliostats connectable per subnetwork and $b_{\text{maxCU}} = 8$ outputs of the LOC of type *branching copper* such that we get

$$k = \lfloor \frac{p_{\text{max}}}{b_{\text{maxCU}}} \rfloor = \lfloor \frac{128}{8} \rfloor = 16.$$

In case of each LOC of type *branching copper* being connected with the maximum amount of b_{maxCU} heliostats in the last step, it is guaranteed that the restrictions of the protocol are satisfied. We thus create a capacitated minimum spanning tree with a capacity restriction of $k = 16$.

Even if we do not have crossings in the layout with the centers of the groups we have to consider the copper cables that will be laid in the next step. If a fiberglas cable is installed between two heliostats that are more than one belt apart from each other we may have crossings in the final layout. Hence we only allow predecessors to be of the same belt or of the belt directly beneath or above. The result can be seen in Figure 19.

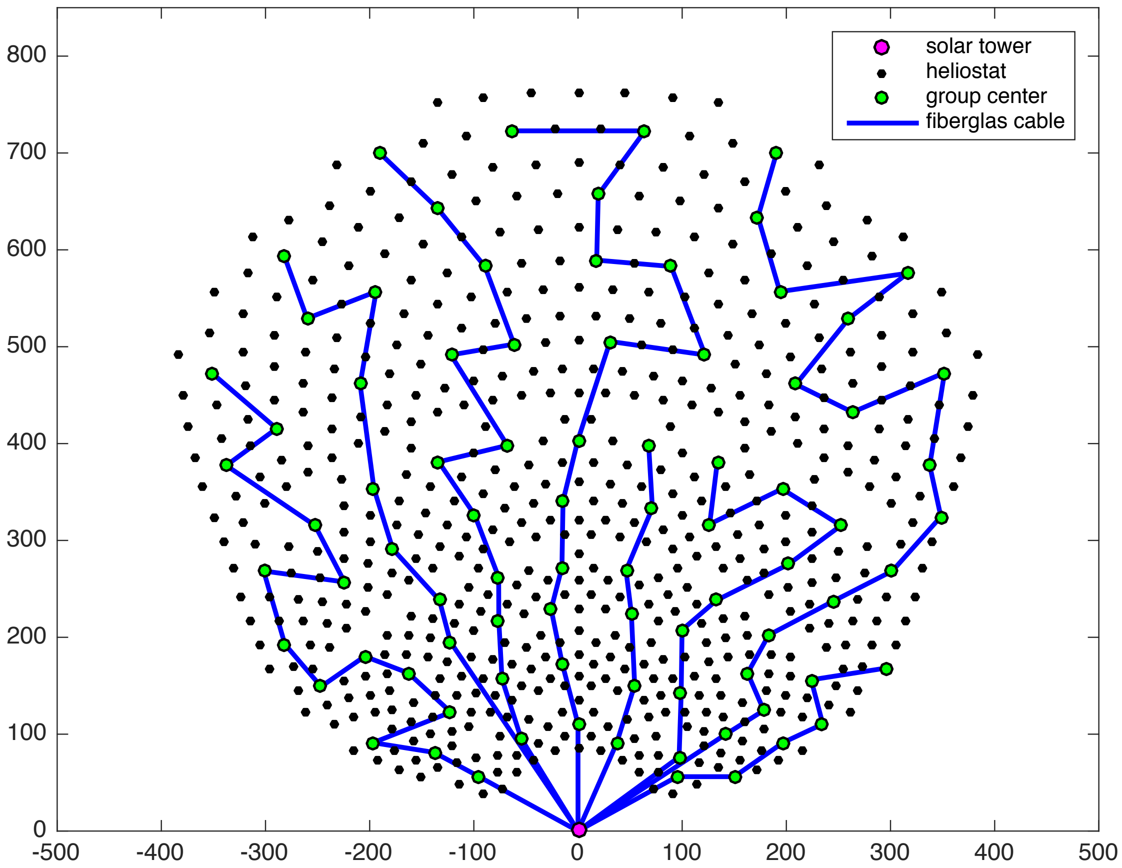
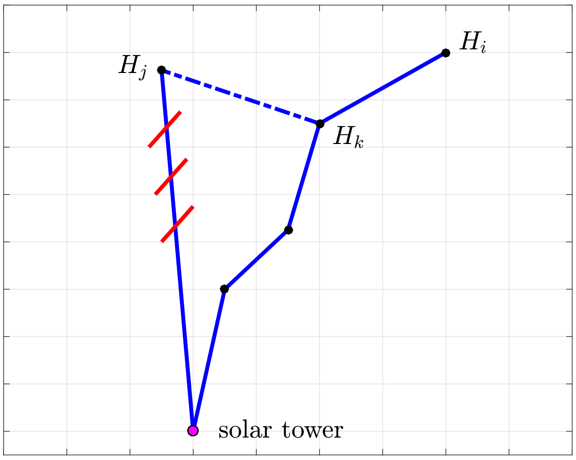


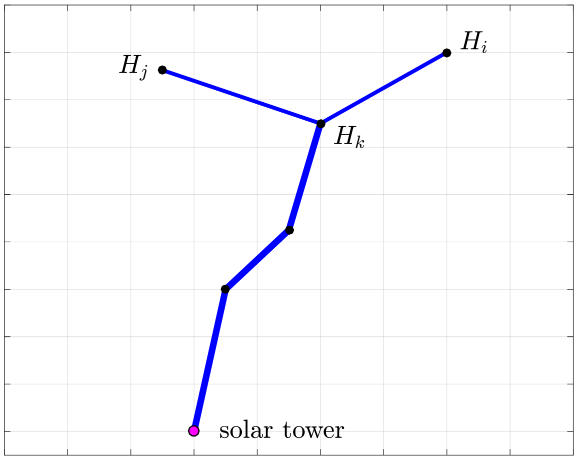
Figure 19: Solar tower: PS10, country: Spain, fiberglas cable length: 6 388.21m, costs for fiberglas cables and switches: 227 258.09€. Applying Esau Williams Version 2 to the centers of the groups. Two connections from belt two and belt three may still cause crossings.

Since there are still some heliostats that are not part of the most inner belt and have the solar tower as predecessor we still may have crossings in the final layout. We cannot implement avoiding this directly in the algorithm since the Esau Williams heuristic always considers the capacity restriction k. If the sum of the heliostats being connected to two wiring looms exceeds k they are not merged so that connections between a heliostat not being placed in the most inner belt and the solar tower may occur. That is why we modify the solution after the heuristic has finished. Basically we allow branching of the fiberglas cable namely that we allow the cables to be installed parallel.

Approach: For all heliostats H_j that are connected to the solar tower and not placed in the most inner belt we search for the heliostat H_k that has the shortest distance to H_j and is placed in the same belt or one beneath or above. The extra costs that have to be taken into account are the total costs for the cable from H_k to H_j and the costs for the cable that follows the way from H_k to the solar tower (see Figure 20).



(a) Crossings may occur if heliostat H_j does not lay in the most inner circle and has the solar tower as predecessor. Instead it shall be connected to the nearest heliostat.



(b) Revised version. Heliostat H_j is connected to H_k instead of the solar tower. The thicker lines represent two cables being laid parallel.

Figure 20: Cables are installed parallel.

This even reduces the total costs because the trench from heliostat H_j to the solar tower is not needed and instead a shorter trench is dug from H_j to H_k. The rest of the cable will be laid into the already existing trench from H_k to the tower. In that case the only additional costs that arise on this route are the cable costs for fiberglas. That is why using switches for branching is not worthwhile in this case.

Altogether the results of Esau Williams Version 2 can be seen in Figure 21.

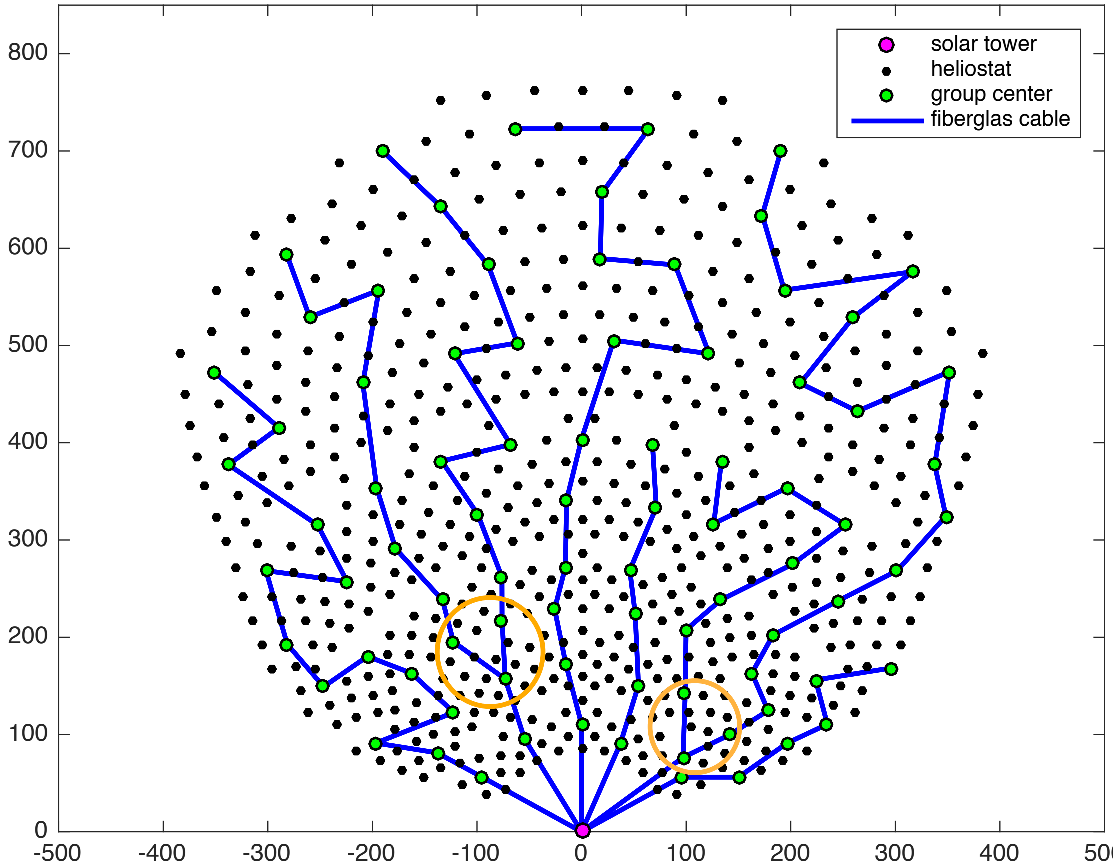


Figure 21: Solar tower: PS10, country: Spain, fiberglas cable length: 6 395.1m, trench length: 6 097.39m, costs for fiberglas cable and switches: 219 419.73€. Applying Esau Williams Version 2 on the computed centers of the groups. New cable connections can be seen in the orange circles. Some cables are installed parallel to avoid crossings of copper and fiberglas cables. Thus trench and cable length differ.

The main thing is that we reduced the runtime significantly. Performing Esau Williams Version 2 only considering the centers of the groups reduces the amount of nodes to approximately an eighth of the original problem. Beside that the use of one type of cable and one switch only accelerates the algorithm so that performing it using Matlab only needs 26.04 seconds. Thus more than 99 % of runtime could be saved compared against Esau Williams Version 1.

Subproblem 3: Connecting the remaining heliostats with copper cables

The final step as already mentioned is to connect the centers of the groups with the other heliostats placed in the same group by a copper cable. The result is displayed in Figure 22.

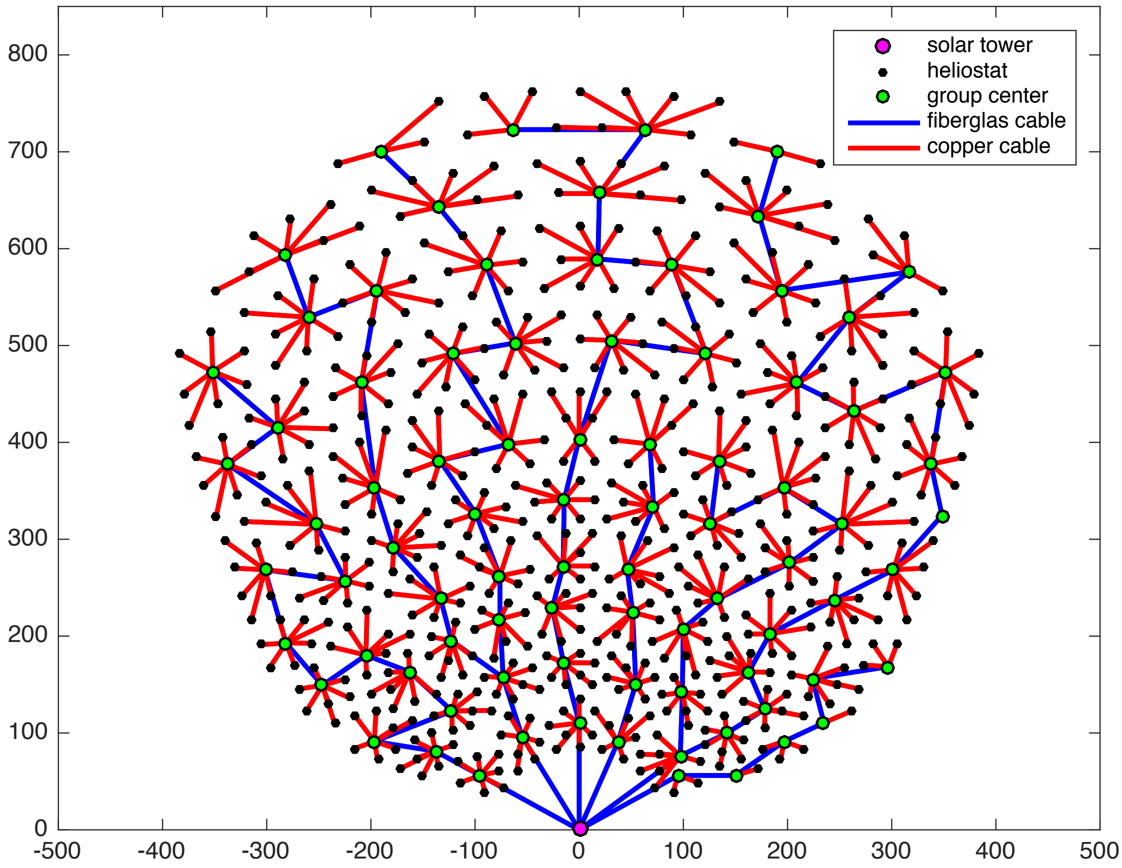
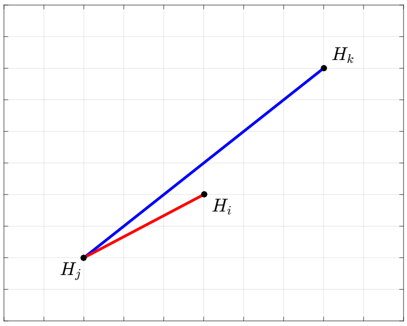


Figure 22: Solar tower: PS10, country: Spain, fiberglas cable length: 6 395.1m, copper cable length: 18 371.98m, trench length: 24 469.37m, total costs: 733 723.58€. Each center is connected with the other group members by a copper cable.

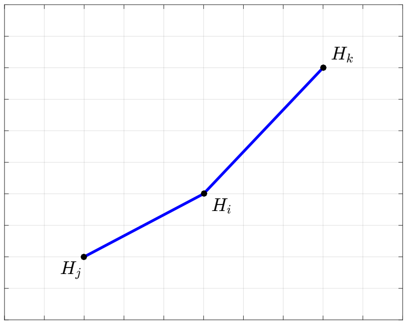
Subproblem 4: Testing if copper connections are sensible or not

While taking a closer look at the layout it is clearly visible that some connections exist that could be avoided to save some additional cable meters. These cases occur when a heliostat H that is connected by a copper cable lies very close to a cable made of fiberglas. Instead of connecting this heliostat separately cable could be saved simply by detouring the fiberglas cable via heliostat H and remove the copper cable completely. The costs for the LOC of type *conductor* instead of *endpoint* have to be taken into account as well. As it is a linear search the runtime is $\mathcal{O}(n)$.

An example can be seen in Figure 23.



(a) Option 1.



(b) Option 2.

Figure 23: Example for cabling that could be replaced by a shorter and more economic alternative. One square in the grid has a side length of 20 meters.

The costs c_1 and the cable length ℓ_1 for the first option are computed by

$$\begin{aligned} c_1 &= c_{\text{totalCU}} \cdot dist_{ij} + c_{\text{totalFG}} \cdot dist_{jk} \\ &= 27.7 \cdot \sqrt{5\,200} + 29 \cdot \sqrt{28\,800} \approx 6\,918.94 \; [€] \end{aligned}$$

$$\begin{aligned} \ell_1 &= dist_{ij} + dist_{jk} \\ &= \sqrt{5\,200} + \sqrt{28\,800} \approx 241.81 \; [m] \end{aligned}$$

and for the second

$$\begin{aligned} c_2 &= c_{\text{totalFG}} \cdot (dist_{ij} + dist_{ik}) - c_{\text{endpoint}} + c_{\text{conductor}} \\ &= 29 \cdot (\sqrt{5\,200} + \sqrt{10\,000}) - 10 + 100 \approx 5\,081.22 \; [€] \end{aligned}$$

$$\begin{aligned} \ell_2 &= dist_{ij} + dist_{ik} \\ &= \sqrt{5\,200} + \sqrt{10\,000} \approx 172.11 \; [m]. \end{aligned}$$

Option 2 saves 1 837.72 € and a cable length of approximately 70 meters.

Adding this feature to the algorithm about 1 700 more cable meters can be saved in the layout for PS10. We chose to detour the connection if the distance between a heliostat connected with a copper cable and the next fiberglas cable is $d = 5$ meters or less. This parameter can be changed individually, however it should not be chosen to be too high because at some point the detoured route will be more expensive than the original connection. The final result can be seen in Figure 24.

This solution exhausts the switches of type *branching copper* very well and thus a lot of LOCs of the economical type *endpoint* are needed. Besides, as few fiberglas cables as possible are used and more of the cheap copper cables are installed instead.

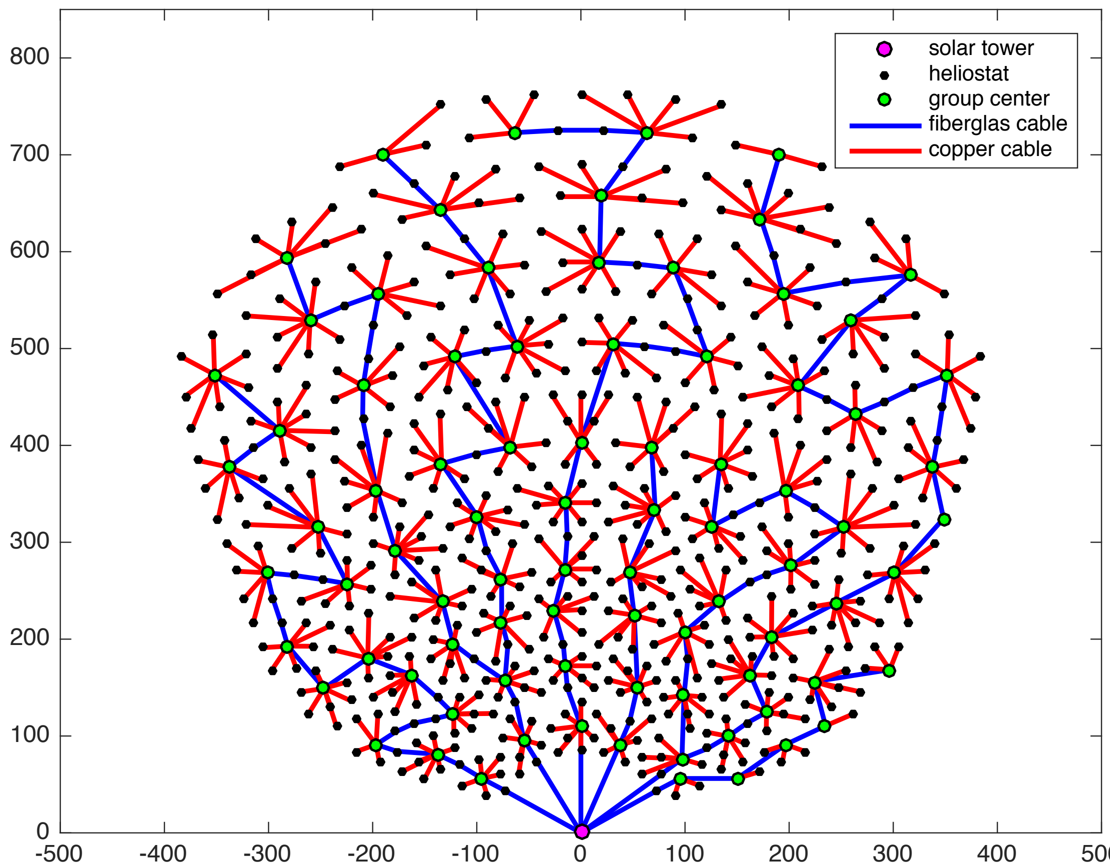
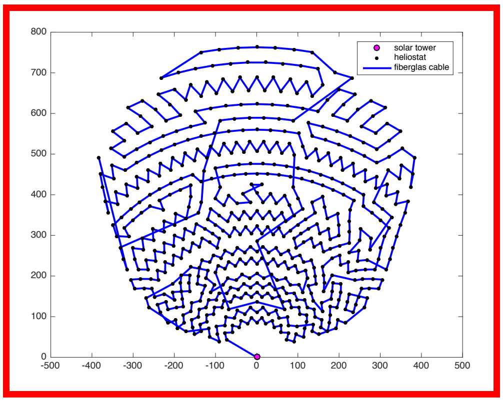


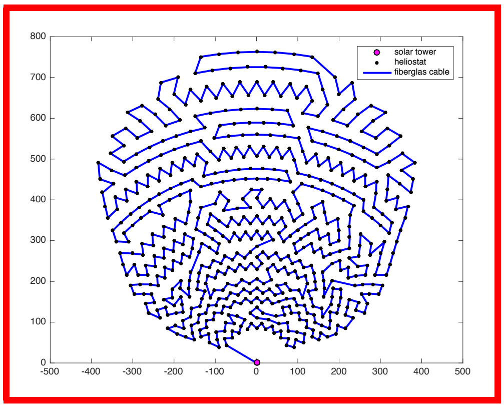
Figure 24: Solar tower: PS10, country: Spain, fiberglas cable length: 6 402.2m, copper cable length: 16 640.8m, trench length: 22 745.29m, total costs: 690 912.38€. Final layout of step 5b. Each group member is connected to the center of its group with a copper cable and not sensible connections are removed.

3.5 Results Overview

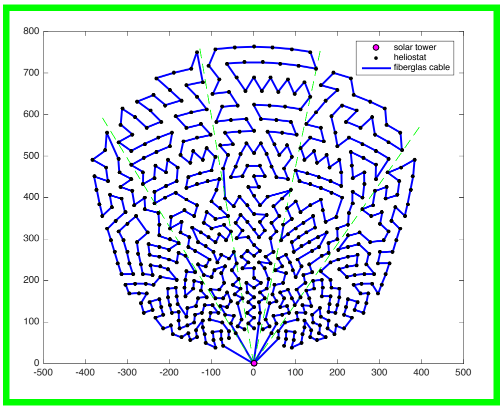
We close Section 3 with an overview of all computed results. The figures having a green frame are valid solutions whereas those with a red one are not valid because they contain crossings or do not respect the restrictions of the used protocol.



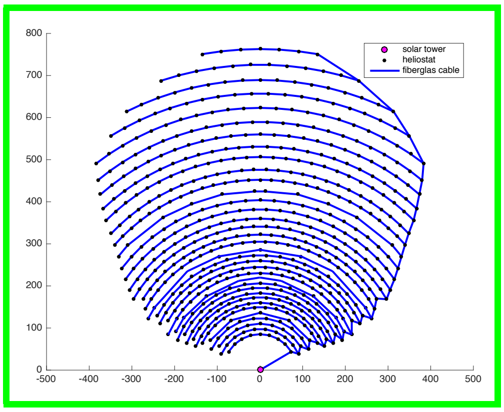
(a) Hamiltonian path.
Total costs: 596 614.63€.



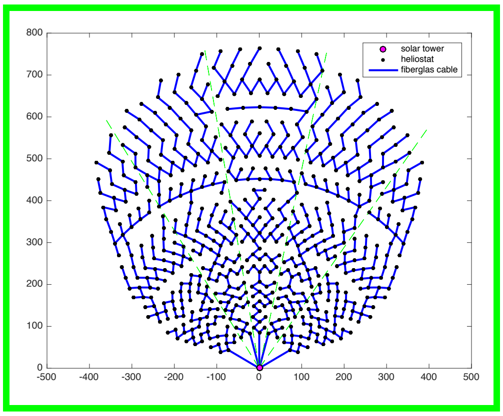
(b) Hamiltonian path without crossings.
Total costs: 548 071.23€.



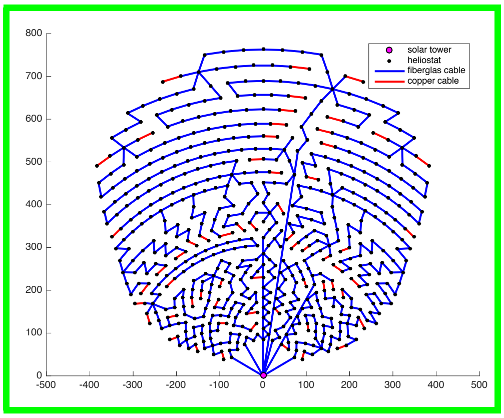
(c) s-Hamiltonian path without crossings.
Total costs: 572 316.59€.



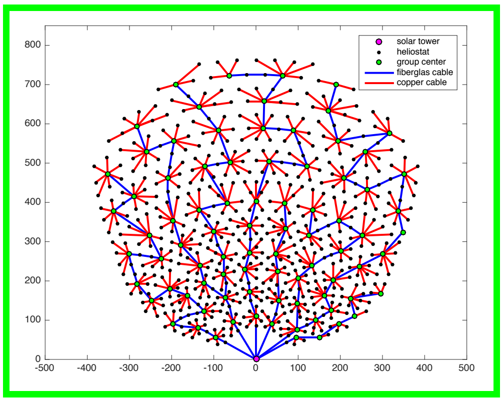
(d) Naive circular pattern.
Total costs: 689 378.44€



(e) Minimum spanning tree.
Total costs: 596 575.82€.



(f) MLCMST computed with Esau Williams V1. Total costs: 623 898.38€.



(g) Groups computed with Esau Williams V2.
Total costs: 690 912.38€

Figure 26: Overview of all computed results and the total costs for placing the power plant in Spain. The green frames mark valid solutions, the red ones are invalid.

4 Comparison of the Results for the Data Cable

In this section we will compare the different results of the algorithms for the data cable that yielded to a valid solution. We will compare the length of the cabling as well as the total costs to be able to recommend one of the applied methods.

4.1 Comparison and Recommendation

We generated valid solutions with the algorithms used in step 3 to 5 and thus compare the quality of the results of these steps only.

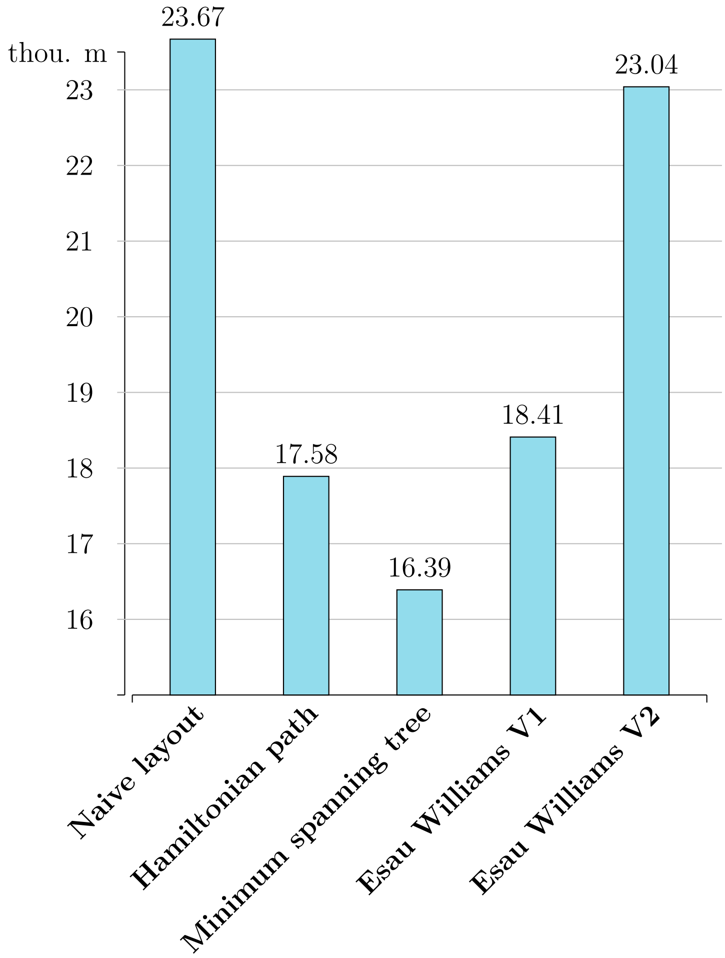


Figure 27: Solar tower: PS10, country: Spain. Comparison of the total cable length of the different results for the data cable in thousand meters.

When taking a first look at the total cable length only as displayed in Figure 27, Kruskal's algorithm is recommended since compared to the naive layout it reduces the total cabling by approximately 30% followed by the s-Hamiltonian path without crossings with about 25% and the result of applying Esau Williams Version 1 with

22%. The solution computed with the help of Esau Williams Version 2 seems like a poor solution since the improvement of the cable meters is with 2.7% very small. However, the total costs not only depend on the cabling but also on the amount and type of switches used in the solar field. As can be seen in Figure 28 a recommendation for one of the presented strategies depends on the country and thus on the payment of the workers the power plant is built in.

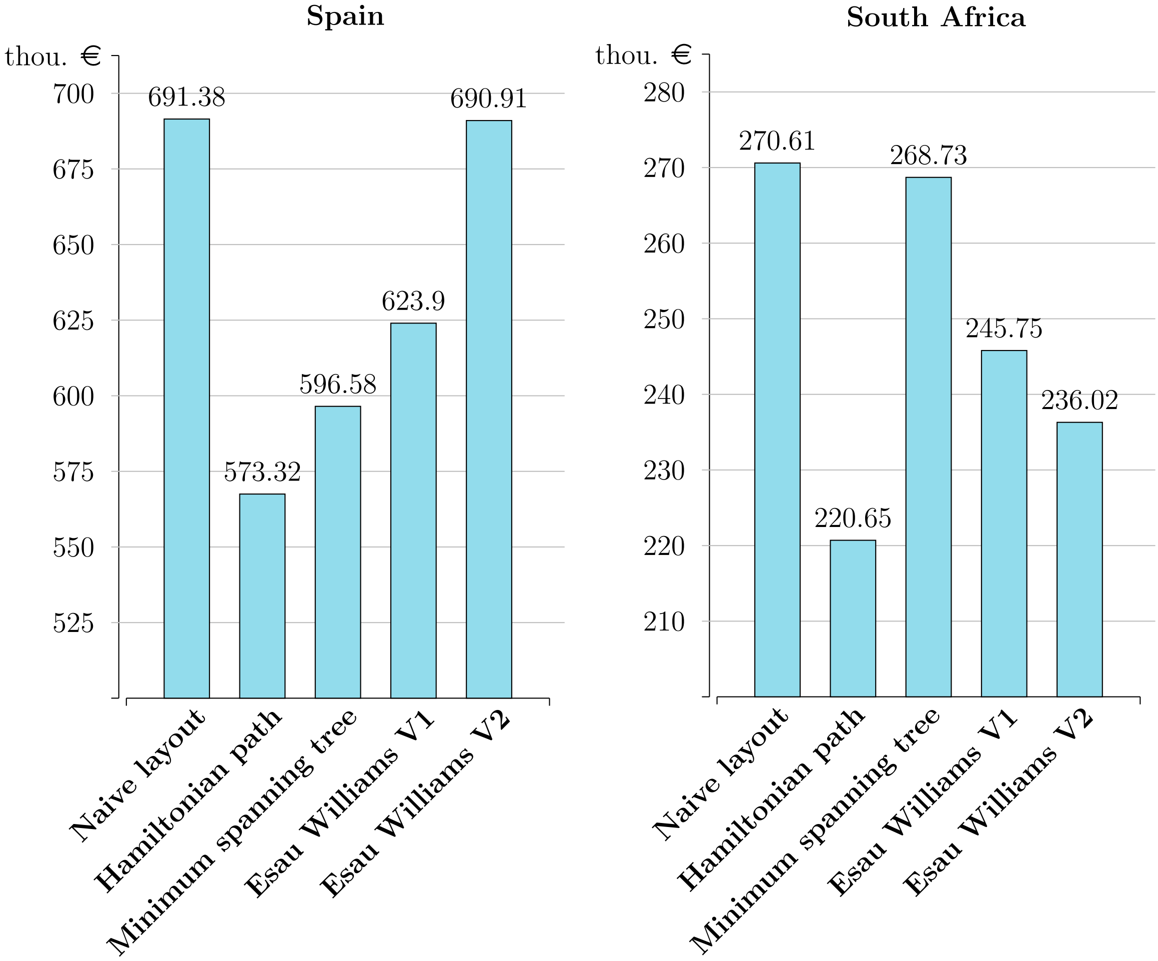


Figure 28: Solar tower: PS10. Comparison of the total costs (in thousand euros) for the data cable dependent on the country the power plant is built in. Note the different axis scaling.

For instance, even if Kruskal's algorithm computes a layout using the fewest cable meters it is not recommended for countries with cheap manpower because it improves the total costs by less than 1% for South Africa. In contrast it is a rather good solution for Spain as it reduces the total costs by approximately 14%. A reason for that is that due to the few cable meters and low cable price per meter the switches carry significantly more weight in South Africa than in Spain.

For the same reason Esau Williams Version 2 is rather recommended for countries with cheap manpower. Even if there is almost no cutback of cable this layout saves approximately 13% of the original costs because of two reasons. On the one hand a lot of cable laid in the field is made of cheap copper and on the other hand this layout uses mostly switches of the economic type *endpoint* and only few of the other two types.

Applying Esau Williams Version 1 to the PS10 heliostat field realizes a cost cutting of 10% in both Spain and South Africa. However, as stated earlier there are better results because a lot of switches of type *conductor* and *branching* are used. By not connecting the full amount of cables to the branching outputs and by generating several long connections there is an in improvement of costs by only 10%.

Ultimately in both cases the most economic layout is created by the s-Hamiltonian path considering the restrictions of the protocol. Since the total cable meters are rather small and because only LOCs of type *conductor* are being used it generates the best solution. It is therefore chosen to be tested for the huge heliostat field in Section 8.

5 Power Cable Model

This section deals with the characteristics of the cables and technology used for powering the heliostats of solar tower power plants. The physics to understand the criterions to choose the right cable type will be explained in detail before we present the constraints and costs that have to be considered in the last subsection.

5.1 Characteristics of the Power Cable

The optimization of the power cable seems to be a more extensive task because we need to distinguish between different cable types and consider their maximal length dependent on how many heliostats are connected to a cable. Thus the price for the cable does not depend on the length of the cable only but also on its cross section.

The conductor is made of copper. Each heliostat is equipped with a motor having a power rating of $P_{\text{motor}} = 100$ watt. A set of cables each having another cross section will be used in our model. The higher the cross section the more heliostats can be provided with power. They have to be thick enough to be able to move all heliostats being connected to it simultaneously.

In contrast to the data cable switches that allow branching are not required. Instead the junction box being installed at every heliostat needs more terminals dependent on how many cables branch off. These additional terminals are low priced in such a way that they will not affect the costs gravely. Hence, we do not include these costs in our model and allow branching without any further increase of costs. Basically an arbitrary number of cables is allowed to branch off a heliostat. In this thesis we confine ourselves to $b_{\max} = 16$ outputs per heliostat.

We do not allow to lay more than one cable into one trench since the cables heat and thus affect their current rating. As a consequence cables would be able to power less heliostats than in case of laying them in separate trenches.

5.2 Physical Background

We need to take care of two things: the cable capacity and the maximum length per cable dependent on the power. To compute the capacities $k_i,\ i \in \{1, ..., n\},\ k_i \in \mathbb{N}$, of the n different cables used in the model we need information about the power P, voltage U, power factor $cos(\phi)$ and the energy conversion efficiency $\eta = \frac{P_{out}}{P_{in}}$.

With these values the electric current I is calculated by

$$I = \frac{P}{U \cdot \cos(\phi) \cdot \eta}. \tag{1}$$

The power P depends on the amount of heliostats connected to a cable. As mentioned in the previous subsection each heliostat has a power of $P_{\text{motor}} = 100$ W. The formula for x heliostats being connected by a cable is thus

$$P = x \cdot P_{\text{motor}} \text{ [W]}.$$

An example for this can be seen in Figure 29.

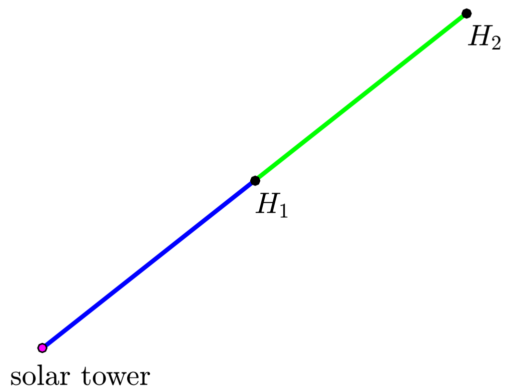


Figure 29: The blue cable powers H_1 and H_2 (200 W in total) while the green one powers H_2 only (100 W).

After examining the capacity one has to determine the permissible length ℓ_i of cable type i dependent on the power that is transmitted. Therefore, the following formula is used:

$$\ell_i \text{ [m]} = \frac{q \cdot \kappa \cdot dU \cdot U \cdot \cos(\phi)}{2 \cdot P}. \tag{2}$$

The parameter κ is the electrical resistivity of copper that is set as $\kappa = 57 \frac{m}{\Omega \cdot mm^2}$, dU is the voltage drop and q the cross section of the cable type.

In the following subsection we will present the used cable types and have a closer look at the calculations of the capacities and maximum permissible cable lengths.

5.3 Cable Types

We ran our program with $n = 7$ different buried cables of type NYY-J 3 x q RE, where q represents the cross section in square millimeters, $q \in \{2.5, 4, 6, 10, 16, 25, 35\}$. The user can add or remove an arbitrary number of cables individually.

Due to the small load we consider a voltage of $U = 230$ V (AC). In our test run we chose the voltage drop to be $dU = 6\%$ that is 13.8 V. Moreover, we chose the power factor $cos(\phi)$ to be 0.95 and the energy conversion efficiency $\eta = 0.9$. These two parameters depend on the motor that is installed at every heliostat. Table 7 gives an overview on the additional parameters that are used in the model.

Each cable type i has a maximum current rating r_i dependent on its cross section as mentioned in Table 6. The current rating is dependent on the laying method and the surrounding temperature. In our case the cables are buried at least 0.8 meters below ground level where a steady temperature of 20°C is assumed. The current ratings can be found in DIN VDE 0298, laying method D, cf. [4].

Based on these characteristics the cable capacity can be calculated with the help of formula (1) of the previous subsection. Since we do not want the cables to use to capacity we include a reserve of 20% meaning that we reduce the maximum current rating to $R_{\max} = 80\%$. This also leads to a slower altering of the cables.

An exemplary calculation for the cable capacity k of cable 3 (NYY-J 3 x 6 RE) with current rating $r_3 = 59$ is shown below.

$$\begin{aligned} \frac{P}{U \cdot \cos(\phi) \cdot \eta} &\overset{!}{\leqslant} r_3 \cdot R_{\max} \\ \Leftrightarrow \quad \frac{k \cdot 100}{230 \cdot 0.95 \cdot 0.9} &\leqslant 59 \cdot 0.8 \\ \Leftrightarrow \quad \frac{k \cdot 100}{196.65} &\leqslant 47.2 \\ \Leftrightarrow \quad k &\leqslant 92.82 \quad \Rightarrow \quad k = 92 \end{aligned}$$

If more than 92 heliostats are connected one has to choose a cable of higher capacity. The other capacities shown in Table 6 are computed the same way.

An exemplary calculation for the maximum length of the cable NYY-J 3 x 6 RE connecting 92 heliostats is shown below.

$$\begin{aligned} \ell_3 &= \frac{q \cdot \kappa \cdot dU \cdot U \cdot \cos\phi}{2 \cdot P} \\ &= \frac{6 \cdot 57 \cdot 13.8 \cdot 230 \cdot 0.95}{2 \cdot 92 \cdot 100} \\ &\approx 56.04\ [m]. \end{aligned}$$

Hence the cable connecting the first of the 92 heliostats has to be shorter than 56.04 meters. If the distance exceeds this value a cable of higher capacity has to be chosen. It is important that we consider that in our model otherwise safety cannot be ensured anymore because in case of failure such as a short protections cannot work reliably anymore.

The second last column of Table 6 denotes the maximal length of each cable type in case of the capacity being exhausted completely. The algorithms that we will use compute the permissible length in each step individually dependent on the amount of heliostats being connected to the cable.

i	Cable type	Current rating r_i [A]	Cable capacity k_i	Maximal cable length ℓ_i [m]	Price c_i [€/m]
1	NYY-J 3 x 2,5 RE	36	56	38.36	0.58
2	NYY-J 3 x 4 RE	47	73	47.08	0.87
3	NYY-J 3 x 6 RE	59	92	56.04	1.24
4	NYY-J 3 x 10 RE	79	124	69.3	1.95
5	NYY-J 3 x 16 RE	103	162	84.87	3.13
6	NYY-J 3 x 25 RM	133	209	102.79	5.19
7	NYY-J 3 x 35 RM	159	250	120.31	6.90

Table 6: Current ratings, cable capacities and costs per meter of the seven different cables. The parameter ℓ_i denotes the maximal length of cable i if the total amount of k_i heliostats is connected to it.

	Parameter	Chosen value
Voltage	U	230 V
Voltage drop	dU	6% (=13.8 V)
Power of one heliostat	P_{motor}	100 W
Power factor	$\cos(\phi)$	0.95
Energy conversion efficiency	η	0.9
Electrical resistivity of copper	κ	$57 \frac{\text{m}}{\Omega \cdot \text{mm}^2}$
Utilization of maximal current ratings	$R_{\max}$	80%
Maximum amount of cable outputs per heliostat	$b_{\max}$	16 outputs
Maximum amount of heliostats per subtree	$k_{\max}$	250

Table 7: Overview and description of the parameters used in the model for the power cable.

5.4 Cost Model

As already mentioned earlier switches are not necessary for the laying of power cables. Consequently the total costs consist of the costs for the cables, protective foil and manpower such as in the model for the data cable. All costs that have to be considered are mentioned in Tables 6 and 8.

Materials affecting the total costs		Price	Parameter
Additional material	Protective foil	2 €/m	c_{foil}
Manpower/ digging	Spain	25 €/m^3	c_{manpower}
	South Africa	5 €/m^3	

Table 8: Additional costs that need to be considered dependent on the country, cf. [8].

Combining all information of this section we draw up the model for the power cable in Table 9. Due to the groundwork done in Section 3 and the fact that branching of cabling does not cause extra costs we will not proceed stepwise considering more and more constraints. We will present three approaches to solve the problem instead, each of them generating a valid solution.

Constraints	**Cost model**
• Each heliostat has to be connected to the solar tower • Each heliostat shall only be connected once • Cables are not allowed to cross • The cable is allowed to branch at the heliostats • Consider the capacity of each cable type and its maximal length (both dependent on the cross section and the amount of heliostats being connected to the cable)	• Cable meters used to connect the heliostats • Price of the cable • Installation costs to connect the heliostats with the solar tower • Distinction between different cable types (this means different cross sections)

Table 9: Cost model and constraints for the power cable.

6 Optimization of the Power Cable

This section deals with the optimization of the power cable. Such as for the data cable we present a naive approach installing the cables along circles around the tower. The second approach is rather naive as well because we minimize the cable meters first and choose the suitable cable type for each connection afterwards. On the contrary all aspects are considered directly by the third approach by using Esau Williams heuristic again. The aim is to create a valid cabling layout using as few cable meters as possible and rather thin and thus cheap cables instead of ones having a high cross section.

Compared to the data cable it makes sense to allow branching from the beginning because we do not have to consider costs for switches. As we have already seen in Section 3 the total cable length is shorter when allowing branching and constructing a minimum spanning tree than creating a Hamiltonian path.

6.1 Approach 1: Naive Circular Pattern

First of all, we want to present a simple solution by installing the power cables along circles around the solar tower. In order to do so we initially proceed in the same way as we did in Subsections 3.2.3 and 3.3.2. Instead of the protocol that limits the attendance number of each subnetwork we now have the maximum capacity of the cable with the highest cross section, in this case $k_{\text{max}} = k_7 = 250$. With PS10 having $h_{\text{PS10}} = 624$ heliostats we get

$$s = \lceil \frac{h_{\text{PS10}}}{k_{\text{max}}} \rceil = \lceil \frac{624}{250} \rceil = 3$$

subnetworks in the heliostat field. We thus divide the field into s sections and choose them to be in the form of cake pieces. Afterwards, the heliostats of the same cake piece having approximately the same distance to the tower are connected in arcs. Besides, each arc is connected with the one below and the one above as can be seen in Figure 30. The total costs add up to 567 979.63€ with a cable length of approximately 20 350 meters.

To mark the different cable types in the plot of the computed layout we use different colors as stated below.

Cable type	1	2	3	4	5	6	7
Color in plot	blue	cyan	green	magenta	red	orange	yellow

Table 10: Colors used in the plot to visualize the different cable types.

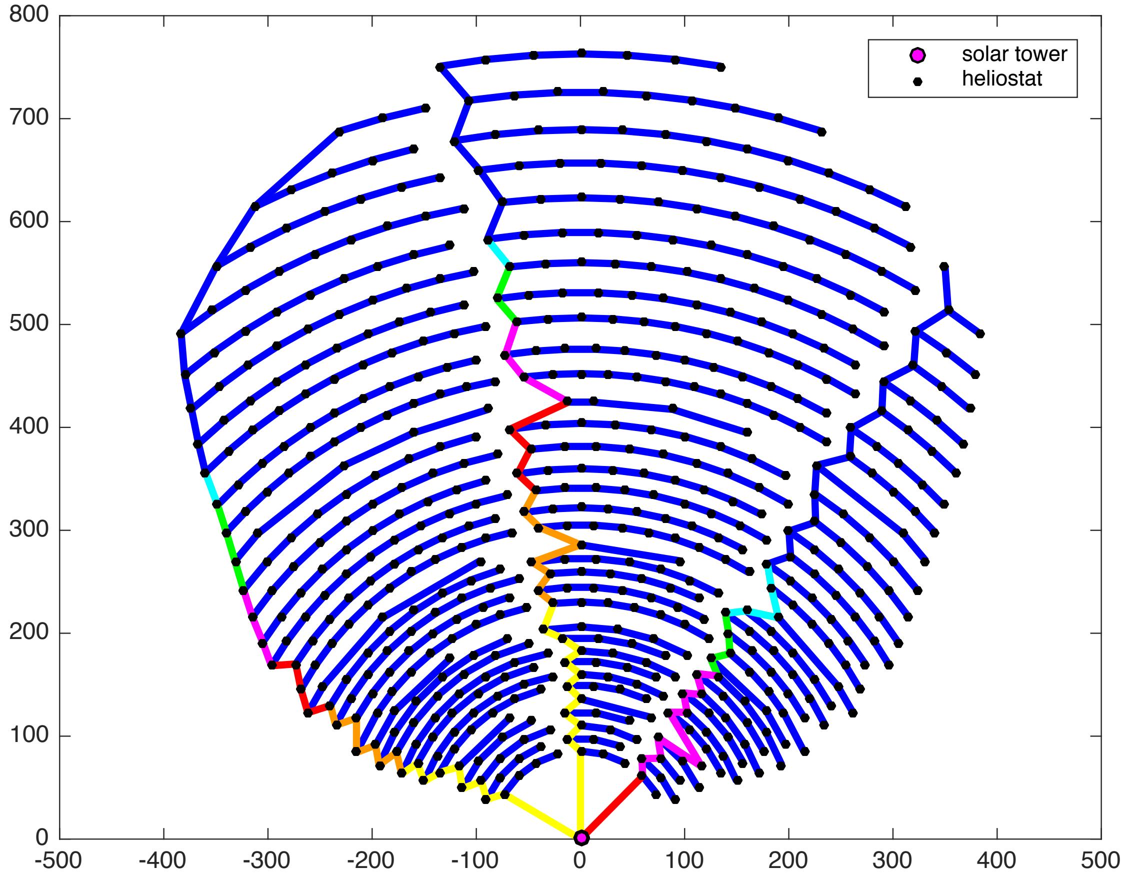


Figure 30: Solar tower: PS10, country: Spain, cable length: 20 351.87m, total costs: 567 979.63€. Naive power cable layout of PS10.

6.2 Approach 2: Minimum Spanning Tree

To improve the first solution for the power cable we use a simple approach meaning that we first minimize the cable meters by creating a minimum spanning tree and choosing the suitable cable type for each connection afterwards.

We initially proceed the same way as described in the previous subsection meaning that we divide the heliostat field into s cake pieces considering the maximum capacity of the power cable with the highest cross section.

In the second step the algorithm constructs a minimum spanning tree for each cake piece with the help of Kruskal's algorithm. At the end for each cable the amount of heliostats that it powers is counted. On this basis every connection is assigned to a cable type dependent on its maximum capacity. For instance, a cable that powers 60 heliostats cannot have a cross section of 2.5 mm^2 but of 4 mm^2 as can be seen in Table 6. Furthermore, the maximal permissible length of the cables dependent on their cross section and the amount of heliostats they power is computed and compared to the actual distance. If the latter is exceeded, a cable of higher capacity has to be chosen.

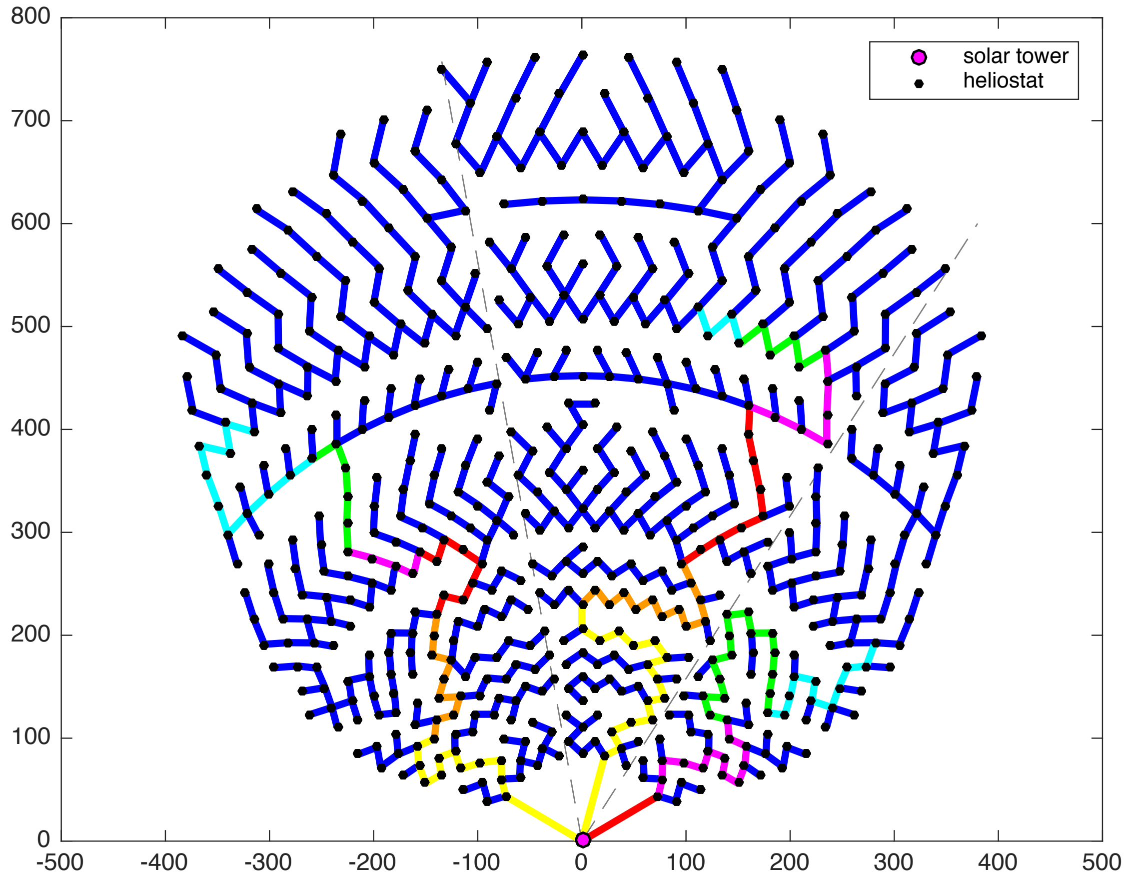


Figure 31: Solar tower: PS10, country: Spain, cable length: 16 214.4m, total costs: 455 647.73€. Three minimum spanning trees connecting each subsection with the solar tower. The cake pieces are indicated by grey dashed lines. Different colors of the cables mark different cable types.

The final layout is presented in Figure 31. Every cable type is represented but mainly cable with the lowest cross section of 2.5 mm^2 is used.

Compared to the naive layout of approach 1 more than 4 000m of cabling and approximately 112 000€ can be saved. However, we want to have another comparison to this solution to be able to say something about the quality of the result. We thus compute another layout with the help of Esau Williams heuristic in the next subsection.

6.3 Approach 3: Multilevel Capacitated Minimum Spanning Tree Computed with Esau Williams Version 3

In contrast to Esau Williams Version 1 and 2 developed for the data cable we do not need to consider costs for switches but costs for cabling only. With the help of Esau Williams Version 3 we construct a multilevel capacitated minimum spanning tree that considers the capacities of the cables as explained in Section 5.3. Besides, in each iteration this version checks if the maximal length of each cable dependent on the amount of heliostats it powers is exceeded.

Moreover, when looking for the cheapest predecessor of a heliostat H the algorithm only considers heliostats being placed near H. The x- and y-value respectively is only allowed to differ $d_{\max} = 70$ meters of the coordinates of H. In that way the algorithm accelerates because only a certain range of heliostats are in line for being predecessor. Heliostats with a higher distance are unsuitable anyway since the costs for laying a cable are higher than choosing a closer placed heliostat.

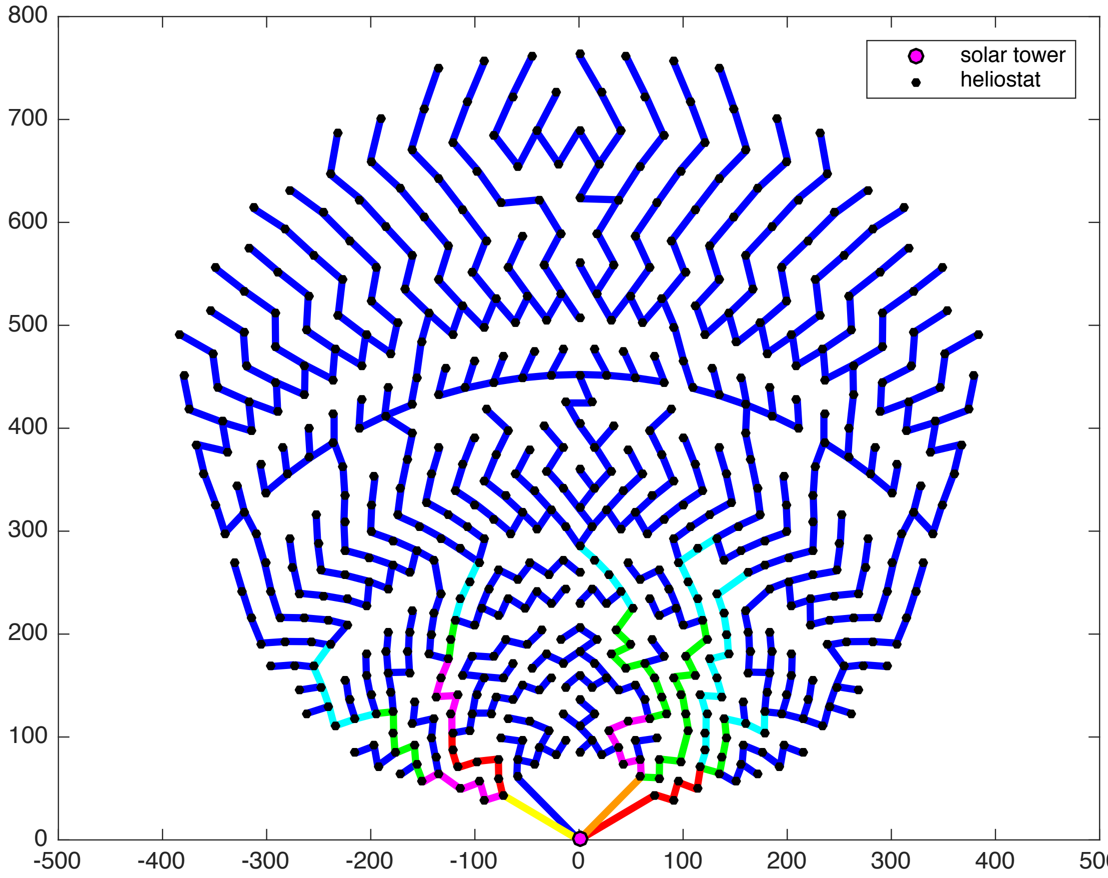
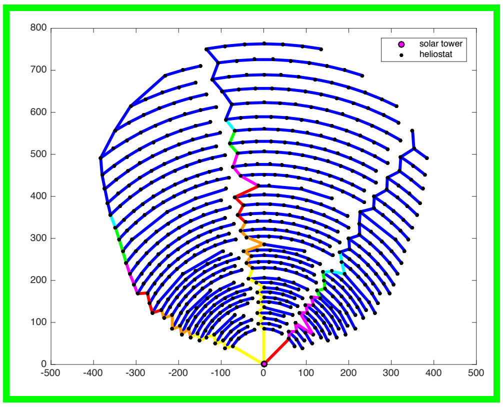


Figure 32: Solar tower: PS10, country: Spain, cable length: 16 388.92m, total costs: 454 665.39€. Result of applying Esau Williams Version 3 to the power cable.

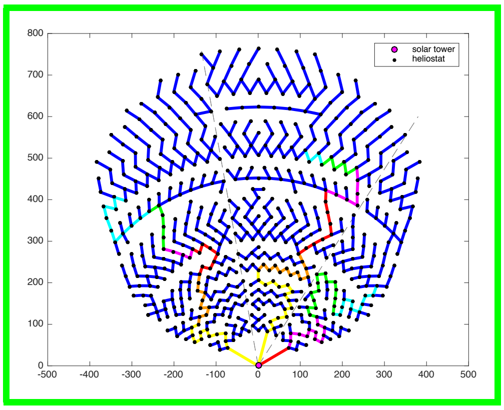
Figure 32 shows that mostly the cheapest and thinnest cable type is installed. In contrast to the minimum spanning tree constructed in approach 2 about 175 more cable meters are being used, however, the total costs are reduced by 982.73 €. This results from less cables having a higher cross section and from not dividing the field into fixed sectors. By doing so we allocate each sector at most $k_{\max} = 250$ heliostats so that in complete groups containing $k_{\max}$ heliostats cable of the highest capacity are needed in any case. Thus Esau Williams heuristic will always lead to a better result, even if the costs differ by less than 1 % to the costs of the minimum spanning tree.

6.4 Results Overview

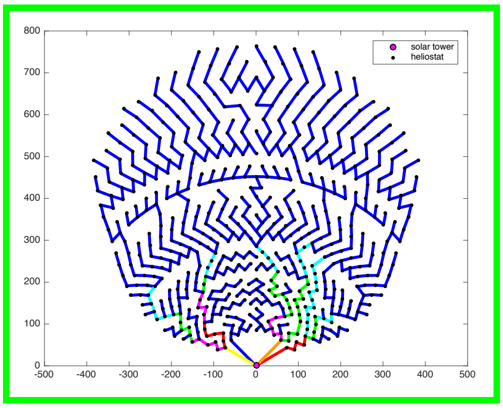
We generated three valid solutions in the previous subsections. Figure 33 gives an overview of the computed layouts and the total costs they cause.



(a) Naive circular pattern.
Total costs: 567 979.63€.



(b) Minimum spanning tree.
Total costs: 455 647.73€.



(c) MLCMST computed with Esau Williams V3. Total costs: 454 665.39€

Figure 33: Overview of all computed results for the power cable and the total costs for placing the power plant in Spain. All solutions are valid.

7 Comparison and Accuracy of the Results

After presenting three strategies for installing power cables in solar tower power plants in the previous section we will now compare them in order to determine the best one relating to the total costs. Furthermore, we will discuss the accuracy of the presented algorithms for the data and the power cable.

7.1 Comparison and Recommendation for the Power Cable

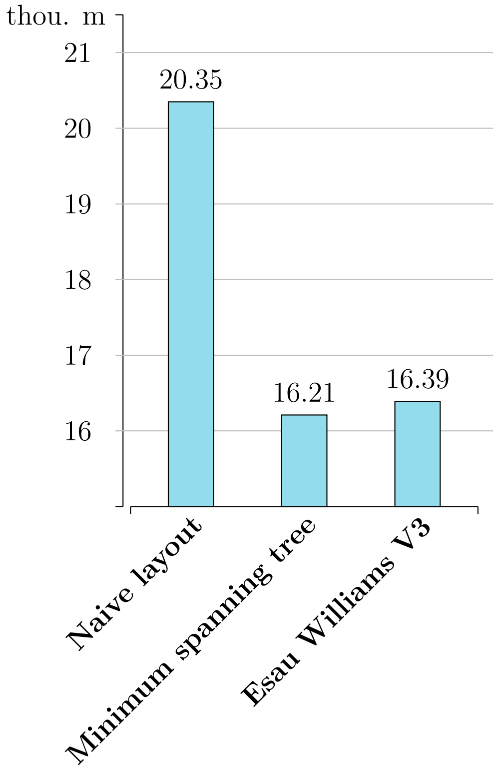


Figure 34: Solar tower: PS10, country: Spain. Comparison of the total cable length of the different results for the power cable in thousand meters.

Figure 34 shows clearly that by applying one of the algorithms presented in Section 6 the total cable meters are reduced by approximately 20% in both cases. This is especially due to branching since in the naive layout for PS10 most heliostats have only one cable that branches off. Besides, there are long connections that could be replaced easily by not laying the cables in circles. Both algorithms not only reduce the cable length significantly but also the total costs as can be seen in Figure 35. Applying Kruskal's algorithm and Esau Williams Version 3 lead to savings of approximately 20% compared to the original layout when placing the power plant in Spain. The savings are a little smaller for South Africa. Kruskal's algorithm saves about 18.4% while Esau Williams Version 3 saves approximately 21%. The higher the installation costs the less influence the costs of the cables are having. Thus the difference between the results of Kruskal's algorithm and Esau Williams heuristic are higher for South Africa than for Spain. Either way Esau Williams heuristic leads to the best result for both countries and thus is chosen to be tested for the huge heliostat field.

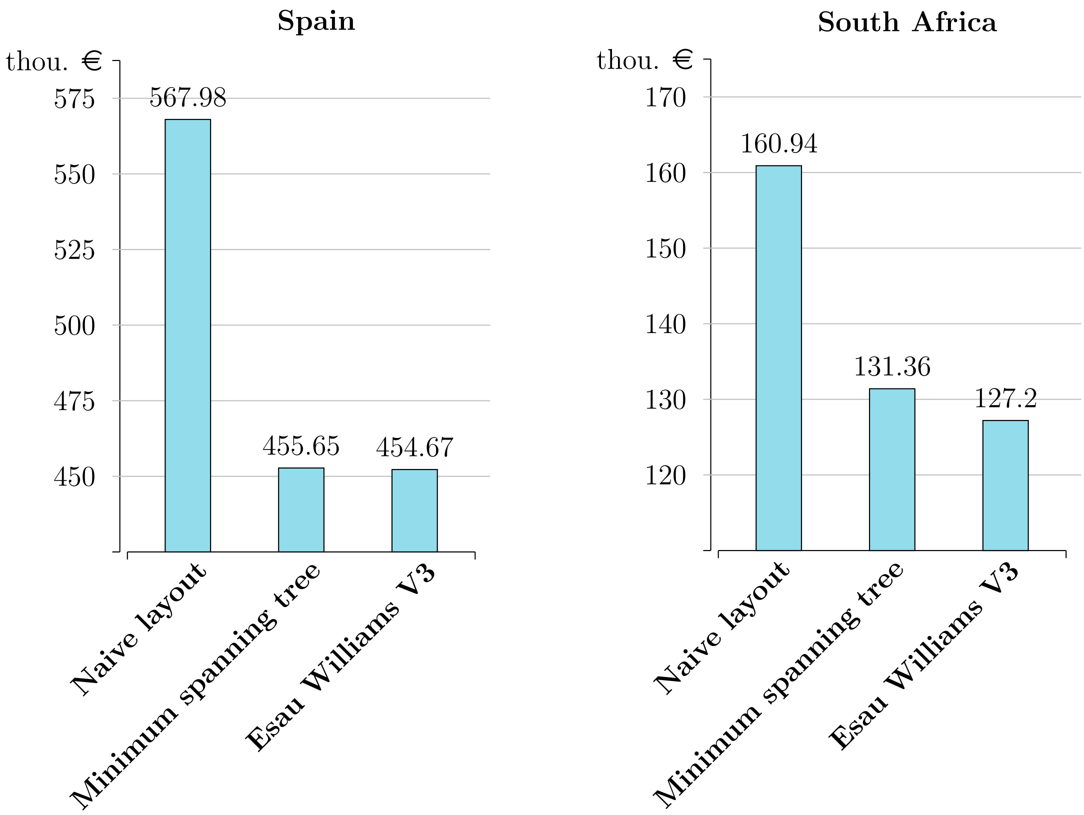


Figure 35: Solar tower: PS10. Comparison of the total costs (in thousand euros) for the power cable dependent on the country the power plant is built in. Note the different axis scaling.

7.2 Accuracy of the Results

The algorithms we presented in this thesis are based on heuristics. The computed layouts thus might not depict the optimal solution, however, using them is worthwhile since we made an improvement of the total costs in almost each case.

To include the restrictions of the used protocol for the data cable and the capacities of the different power cables we divided the heliostat field into cake pieces and computed a Hamiltonian Path and a minimum spanning tree respectively for each of them. By making use of those sections we might impair the solution because a heliostat H_i being the nearest heliostat to H_j might be placed in another sector and thus is not in line for being connected to H_j even if it was the cheapest connection. As a result there might be a segmentation that generates even better results. However, with each subtree starting from the solar tower a good solution will create a layout being roughly in the form of cake pieces. This is because the cable has to reach the heliostats being placed in the outer area and has to connect as many heliostats as possible on its way. Further investigation is recommended.

As stated in [12] there is one major problem with the Esau Williams heuristic namely when all subtrees formed over the course of the algorithm contain more than $\frac{k_{\max}}{2}$ nodes. In this case it is not possible to merge the subtrees since doing so would exceed the maximum cable capacity. Thus each subtree will be connected to the root node r and the algorithm terminates. Figure 36 shows an example of this problem. While the optimal solution only contains two subtrees applying Esau Williams heuristic leads to three subtrees and therefore to higher final costs.

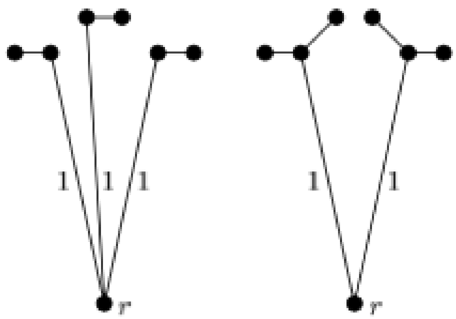


Figure 36: The left-hand side shows the result of applying Esau Williams heuristic to an exemplary graph while the right-hand side shows the optimal solution. Cable capacities $k \in \{1, 2, 3\}$, $k_{\max} = 3$, are being used. Instead of the optimal solution containing two subtrees the solution generated with the help of Esau Williams algorithm creates three.

Comparing the results of the three approaches for the power cable in Section 6 we can see that $s = 3$ subtrees are needed to connect every heliostat and consider the cable capacities (see Subsection 6.1). Applying Esau Williams Version 3 in Subsection 6.3 leads to four subtrees starting from the solar tower. However, the total costs are cheaper than the costs for the minimum spanning tree since less cable with a high and more with the lowest cross section are being used.

To improve the solution it is suggested to give priorities to subtrees dependent on their weight, cf. [12]. The higher the weight the higher the priority so that subtrees with an increased weight grow up to capacity $k_{\max}$ and thus almost equal subtrees with weight of $\frac{k_{\max}}{2}$ are avoided. It is a debatable point whether including this feature to the algorithm is worthwhile because when reducing the amount of subtrees to only s it is likely that more expensive cables with higher cross sections are needed. That is why we quit adding this feature to our algorithm.

8 Case Study

After examining the algorithms that compute the most economic layout for the data and the power cable we apply both to the huge heliostat field. Due to the size of the field we relinquish plotting the results and list them tabularly instead. We ran the algorithms on the same computer we tested Esau Williams Version 1 on before. It has a high computing capacity and is therefore faster in computing the solution. The algorithms are tested with the costs for placing the heliostat field in Spain.

8.1 Data Cable

Since the amount of heliostats in the huge field is more than twenty times higher than in PS10 we adjusted the parameters for the protocol before applying the algorithm. Instead of choosing $p_{\max} = 128$ we extended the used protocol to $p_{\max} = 2\,600$.

Note that we assume the layout to be similar as for PS10 meaning that the field is divided in sections and the cables being installed in circles around the tower. For the data cable we have

$$s = \lceil \frac{h_{\text{DLRField}}}{p_{\max}} \rceil = \lceil \frac{12\,676}{2\,600} \rceil = 5$$

cables altogether starting from the solar tower. Each one connects maximum $p_{\max} = 2\,600$ heliostats that are placed in the same area. They are in the form of cake pieces containing a trench that runs from the tower to the most outer heliostat circle of the field. For each circular segment on its way there is a wiring loom branching off to connect the heliostats of exactly this segment. We only lay one cable into one trench. A piece of the assumed layout is exemplary shown in Figure 37.

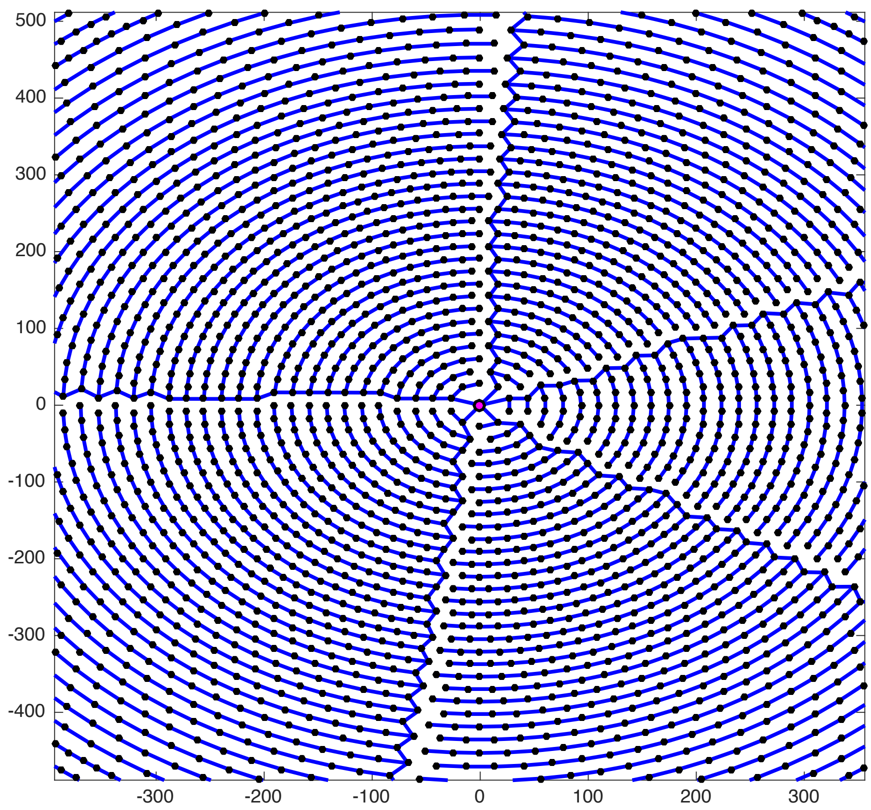


Figure 37: Solar tower: DLR Field, country: Spain. A segment of the naive layout of the data cable for the DLR Field.

	Naive Layout	***s*-Hamiltonian path**
Trench length	298 279.71 m	296 417.94 m
Cable length	298 279.71 m	296 417.94 m
LOC *endpoint*	0 pieces	0 pieces
LOC *conductor*	12 323 pieces	12 676 pieces
LOC *branching fiberglas*	353 pieces	0 pieces
LOC *branching copper*	0 pieces	0 pieces
Total costs	10.0589 M €	9.8627 M €

Table 11: Comparison of the characteristics of the naive layout for the data cable and the layout computed with the help of the shortest distance and 2-opt algorithm.

Against all odds, there is no significant cost reduction when applying the algorithm to the huge heliostat field creating a s-Hamiltonian path. Costs could be saved by using LOCs of type *conductor* only, however, not more than 195 191.40 € could be saved in total. Taking a closer look at the computed layout (see Figure 38) and comparing it with the naive one it becomes a bit more explicit why almost no costs could be saved. The heliostats that are placed more than 580 m away from the tower are mainly connected in circles around the tower as well so that both layouts do not differ significantly in this area of the field. This is because the heliostats being far apart from the tower laying next to each other in the same circle – that means these heliostats have approximately the same distance to the solar tower – are placed very close to one another. These distances are always at least 30% smaller than the distances between the individual circles. This leads the algorithm to connect the heliostats of the same circle rather than generating many connections that connect two heliostats of different circles. Because of the segmentation of the field in cake pieces and creating a Hamiltonian path for each of them mainly two cables connect heliostats of two different circles.

Thus one can say that the chosen algorithm is rather suitable for heliostat fields where the mirrors are evenly distributed such as in PS10 or in the area of the huge field where heliostats have a smaller distance than 580 m to the tower. It may be worthwhile to combine this algorithm with the naive layout for the DLR Field. That means applying the algorithm to heliostats being close to the tower and to connect the other heliostats in circles around the solar tower.

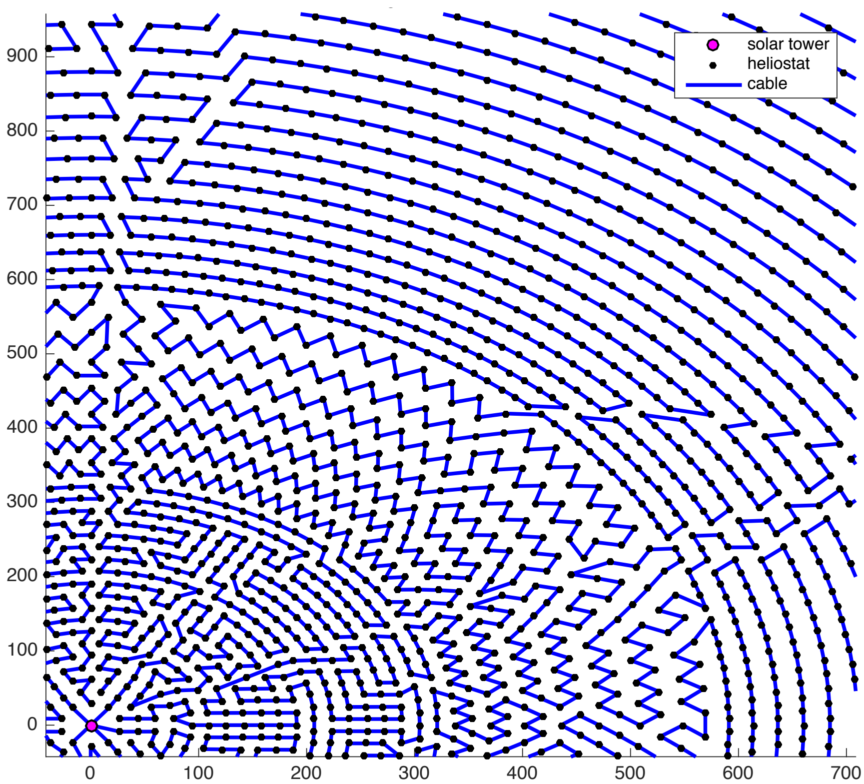


Figure 38: Solar tower: DLR Field, country: Spain. A segment of the optimized layout with the help of the shortest distance algorithm and the 2-opt heuristic.

8.2 Power Cable

Due to the heliostat field having a width of about 3 500 m and a height of about 4 000 m we decided to make use of thirteen power cable types in total. As a consequence less cables start from the solar tower and more heliostats can be connected to them. We expand the seven cables presented in Section 5.3 in Table 6 by six cables having an even higher cross section. Both these cables and their characteristics are stated in Table 12. Again, the current rating r_i can be abstracted from DIN VDE 0298, laying method D, cf. [4]. Cable capacity k_i and maximal cable length ℓ_i is calculated with the help of the formulas presented in Section 5.2.

Cable type	**Current rating** r_i **[A]**	**Capacity** k_i	**Maximal cable length** ℓ_i **[m]**	**Price** c_i **[€/m]**
NYY-J 3 x 50 SM	188	295	145.65	9.55
NYY-J 3 x 70 SM	232	364	165.26	13.16
NYY-J 3 x 95 SM	280	440	185.54	17.85
NYY-J 3 x 120 SM	318	500	206.24	22.14
NYY-J 3 x 150 SM	359	564	228.55	27.53
NYY-J 3 x 185 SM	406	638	249.18	34.13

Table 12: Overview of the additional power cables used for the huge heliostat field. The capacity and maximal cable length is computed with the information about their current rating and the formulas presented in Section 5.2.

In the naive layout the power cables are installed in circles around the tower similar to the data cable but respecting the maximal lengths and capacities of the thirteen power cables. In contrast to the data cable the field is divided in

$$s = \lceil \frac{h_{\text{DLRField}}}{k_{\max}} \rceil = \lceil \frac{12\,676}{638} \rceil = 20$$

segments. An exemplary section of the field is presented in Figure 39.

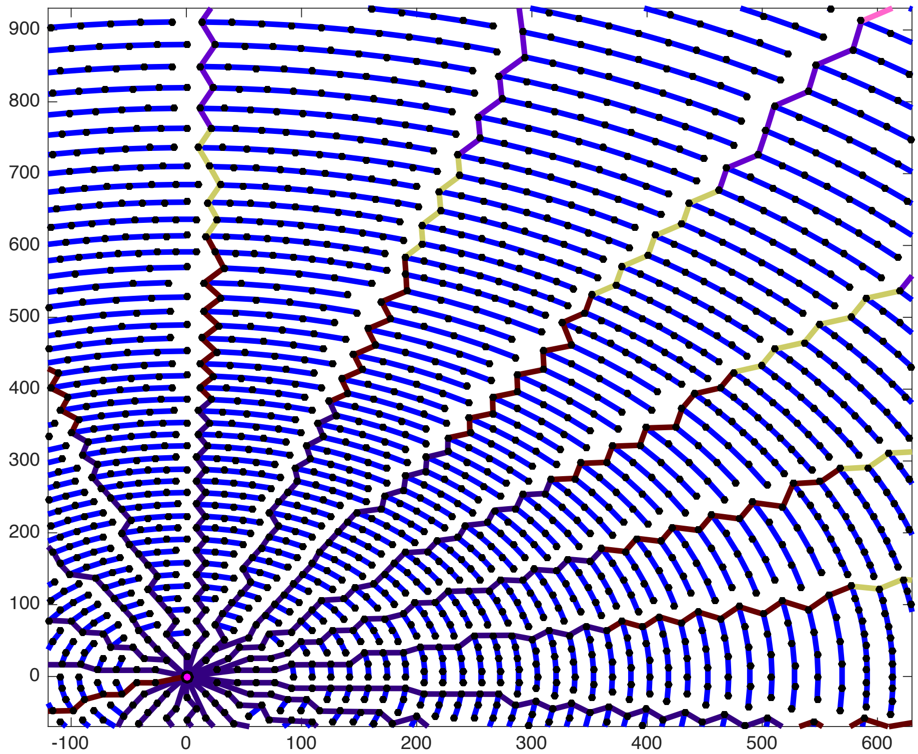


Figure 39: Solar tower: DLR Field, country: Spain. A segment of the naive layout of the power cable for the DLR Field. Different colors mark different cable types.

In Section 7 we determined Esau Williams Version 3 to be the best algorithm for computing an optimal layout for the power cable with the help of heuristics. When running this algorithm with the data of the DLR Field it appeared very quickly that it is not suitable for high amounts of data.

Starting the simulation on the better computer we noticed that the optimization of the first heliostats needed more than 5 minutes per mirror. Even if the algorithm accelerates the more heliostats have been considered, waiting for it to end would still have taken several days due to the amount of 12 676 heliostats.

Also, dividing the field in different areas would still have taken various days to compute the solution so that we decided to fall back on approach 2 presented in Subsection 6.2. This algorithm has a short runtime and – at least for PS10 – generated solutions that only differed by a few percentage points from the layout computed with Esau Williams Version 3.

The cable length and costs arising for the naive layout are compared with the results obtained by applying Kruskal's algorithm to the heliostat field in the following table. Note that for the power cable the trench length is always the same as the cable length since we do not allow to lay more than one cable into a trench.

	Naive Layout	**Minimum spanning tree**
Cable length	305 897.33 m	297 733.39 m
Total costs	9.1022 M €	9.6233 M €

Table 13: Characteristics of the naive layout for the power cable versus the ones of the computed layout with the help of Kruskal's algorithm.

Indeed, by applying Kruskal's algorithm for creating a minimum spanning tree the total cable length is reduced by 8 163.94 m, however, the total costs rise by more than half a million euro which is a very poor result. There are mainly two reasons for that. On the one hand we have the same situation as for the data cable meaning that the heliostats being placed more than 580 m apart from the solar tower are connected with their neighbors of the same arc. This can be seen in an exemplary segment of the computed layout in Figure 40. On the other hand each arc is connected by a cable with the arc laying above and below. Since these connections do not lay at the edge of each cake piece but arbitrary somewhere in between the arcs, there are many connections that are made of cable of a higher cross section. In Figure 40 this can be seen especially by the cables colored in yellow, purple and pink. This is because we apply Kruskal's algorithm first creating a minimum spanning tree and assign each connection the suitable cable type afterwards. In case of laying the cables such as in the naive layout, only the cables connecting the different arcs have to be made of cables with a higher cross section than 2.5 mm^2. Thus this solution will always lead to a more economic result for heliostats placed more than 580 m away from the tower.

We recommend to do the same as for the data cable meaning to optimize the part of the field with evenly distributed heliostats with the help of the presented algorithm. It makes a lot of sense to connect the other heliostats along circles around the tower such as in the naive solution. In that way most connections are made of the cheapest cable type.

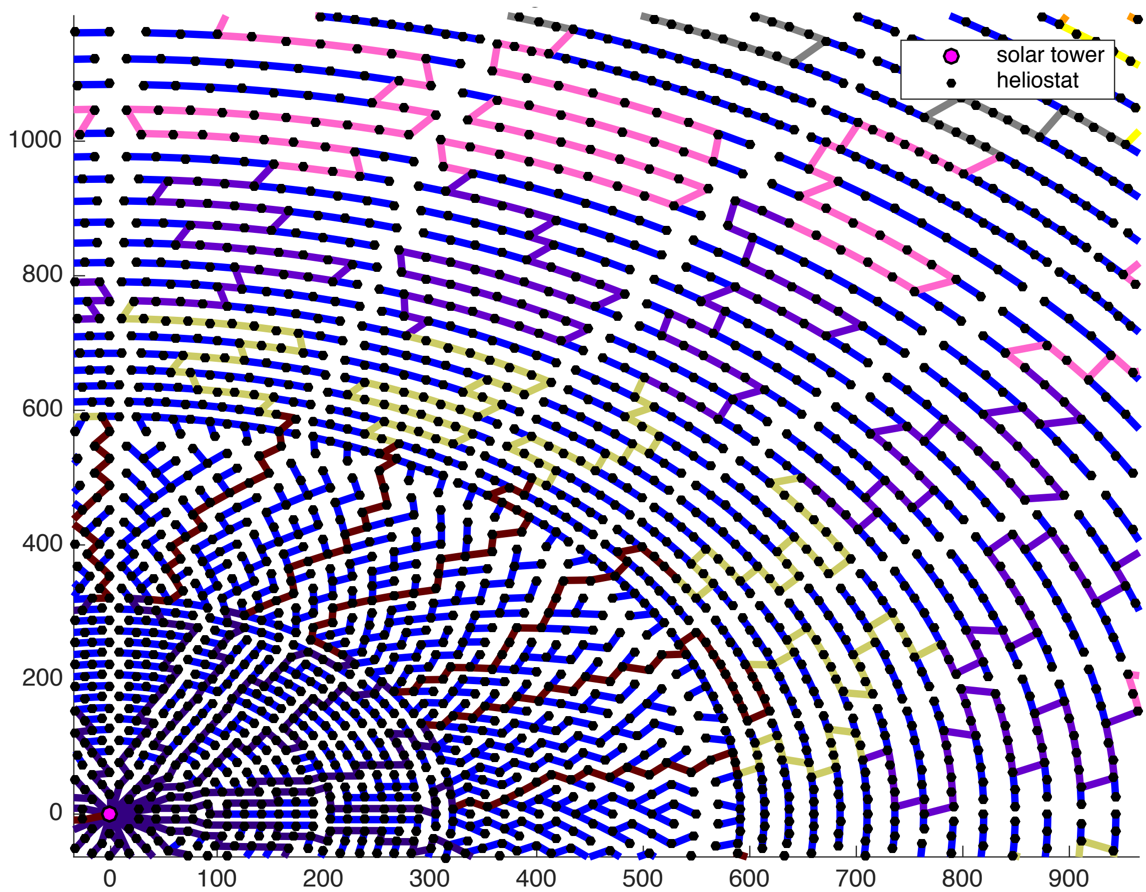


Figure 40: Solar tower: DLR Field, country: Spain. A segment of the layout of the power cable for the DLR Field generated by Kruskal's algorithm presented in Subsection 6.2. Different colors mark different cable types.

9 Conclusion and Outlook

In conclusion using heuristics to optimize cabling in solar tower power plants for small heliostat fields and fields with evenly distributed heliostats is worthwhile. Both the cable length and the total costs of the power and data cable for PS10 could be reduced significantly so that further research in this field is highly recommended. The best solution for the cabling of the DLR Field was created by applying the naive approach and connecting the heliostats in cake pieces along circles around the solar tower. Combining a naive layout for unevenly distributed heliostats with an optimization of the heliostats being placed near the solar tower might be worthwhile.

During the development and implementation of the algorithms some additional features came into our mind that could be added to the model. Since this would have gone beyond the scope of this bachelor thesis we list them now and left including them for future work.

Data cable

- We have been considering four types of LOCs in our model. It may be worthwhile to include another available LOC having one copper cable as input and one copper cable as output. The use of fiberglas may be further limited and the total costs further reduced.

Power cable

- When computing the solution for the power cable for the huge heliostat field we chose the solar tower as root node only. As the field starting from the tower trends to approximately 1 500 m to east and west and about 2 000 m to north and south one should contemplate to divide the field in areas and place power distributors centrally that power the heliostats of the same area. Otherwise, there are plenty of cables starting from the solar tower or an increased amount of expensive cables with a high cross section has to be used. The algorithm presented in Subsection 3.4.3 dividing the heliostat field into groups could be used to do so. Instead of groups of $b_{\mathrm{maxCU}} = 8$ to match a cable to each output of the switch of type *branching* for the data cable one could create much bigger groups containing for example 500 heliostats. In each group a power distributor could be placed centrally to power the heliostats of the same group. In the next step one should perform Esau Williams Version 3 for each group individually with the root node being the power distributor for groups being placed far away from the solar tower or the tower itself for the heliostats being close to it. This would accelerate the runtime of the algorithm because each group containing only a fraction of heliostats of the whole field is being optimized individually. The additional costs for power distributors have to be included.

Features for both cable types

- We created good solutions and cut the total costs for PS10. However, since we use heuristics the next step would be to optimize the cabling by using optimization solvers. With the help of integer linear optimization the optimal cabling could be computed and compared to the solutions we created with the help of heuristics. After doing so one can say something about the quality of our results and one of the approaches may be recommended. In another thesis optimizing the cabling in offshore wind farms it was shown that the results of heuristics do not differ more than 5% from the solution gained by optimization solvers, cf. [3].

- Up to now we treated the optimization of power and data cables as separate problems. Since both have to be laid into the field it is recommended to consider them as one cable that has to fulfill the constraints of both cable types. Costs for trenches and protective foil affect the cost model only once. Besides, the amount of heliostats N per subnetworks is limited by

  $$N \leqslant min\{p_{\max}, k_{\max}\}.$$

  Additional costs that occur in this case are costs for sand used as filling material. One of the cables is laid on the ground of the trench and after covering it with sand the other cable and protective foil is laid above. The rest of the trench is then filled with the soil that has been trenched before.

- Compared to the other algorithms the runtime of Esau Williams heuristic for all three presented versions is with several hours very high. We noticed that this is due to the part that eliminates crossings. Removing it from the code the result for PS10 is computed in less than a minute containing a handful crossings. We recommend to revise this part of the code to warrant a faster runtime. To do so one could exclude certain connections right away by having a look at the coordinates. Figure 41 shows that connections between heliostats that are both placed in the same sector m, $m \in \{1, 2, 3, 4, 6, 7, 8, 9\}$ will not cross the possible new connection from heliostat H_j to H_k. Instead only connections between heliostats not being placed in the same area can lead to crossings. All possibilities have to be considered. For instance there may be one crossing if a connection goes from a heliostat placed in sector 1 to a heliostat placed in sector 5, 6, 8 or 9. These should be examined in further detail.

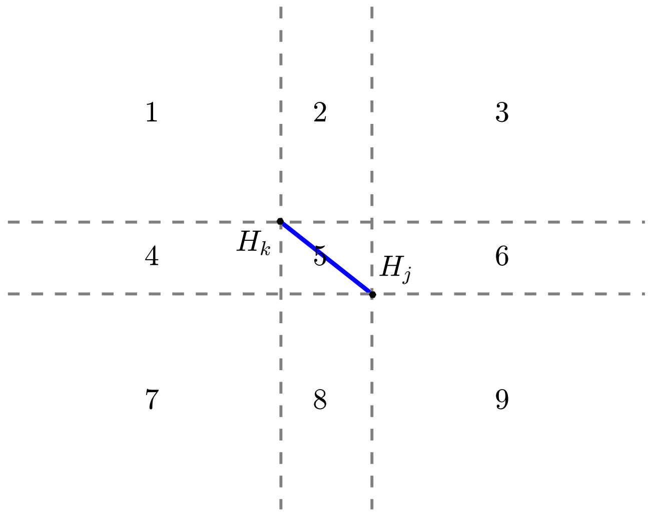


Figure 41: This graphic shows which connections have to be examined in further detail to check for potential crossings. Connections between two heliostats that are placed in the same sector m, $m \in \{1, 2, 3, 4, 6, 7, 8, 9\}$ will not cross with the possible new connection from heliostat H_j to H_k.

- When applying Esau Williams heuristic it may be worthwhile to start with another initial solution. Instead of each heliostat being connected to the solar tower in the first step one could connect the heliostats along circles around the tower. In this way the algorithm will create a solution that combines the naive layout for unevenly distributed heliostats with an optimization of the heliostats being placed near the tower.

- Last but not least the segmentation of the field in cake pieces for considering the protocol for the data cable and the maximum cable capacity for the power cable might not be optimal as already stated in Subsection 7.2. An idea to improve the layout might be to use cake pieces with changeable borders so that two heliostats having the shortest distance to each other but are placed in different sectors can be connected nonetheless.

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